

Complexity of Barrier Coverage with Relocatable Sensors in the Plane^{*}

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Abstract. We consider several variations of the problems of covering a set of barriers (modeled as line segments) using sensors so that sensors can detect any intruder crossing any of the barriers. Sensors are initially located in the plane and they can *relocate* to the barriers. We assume that each sensor can detect any intruder in a circular area centered at the sensor. Given a set of barriers and a set of sensors located in the plane, we study three problems: the feasibility of barrier coverage, the problem of minimizing the largest relocation distance of a sensor (MinMax), and the problem of minimizing the sum of relocation distances of sensors (MinSum). When sensors are permitted to move to arbitrary positions on the barrier, the problems are shown to be NP-hard. We also study the case when sensors use *perpendicular* movement to one of the barriers, thereby moving to the closest point on the barrier. We show that when the barriers are parallel, both the MinMax and MinSum problems can be solved in polynomial time. In contrast, we show that even the feasibility problem is NP-complete if two perpendicular barriers are to be covered, even if the sensors are located at integer positions, and have only two possible sizes. On the other hand, we give an $O(n^{3/2})$ algorithm for a natural special case.

1 Introduction

The protection of a region by sensors against intruders is an important application of sensor networks that has been previously studied in several papers. Each sensor is typically considered to be able to sense an intruder in a circular region around the sensor. Previous work on region protection using sensors can be classified into two major classes. In the first body of work, called *area coverage*, the monitoring of an entire region is studied [1–3], and the presence of an intruder can be detected by a sensor anywhere in the region, either immediately after an appearance of an intruder, or within a fixed time delay. In the second body of work, called *barrier coverage*, a region is assumed to be protected by monitoring its perimeter called barrier [4–8], and an intruder is detected when crossing the barrier. Clearly, the second approach is less costly in terms of the number of sensors required, and it is sufficient in many applications.

^{*} Research supported by VEGA 2/0136/12, NSERC Discovery, and SEP-CONACYT grants.

There are two different approaches to barrier coverage in the literature. In the first approach, a barrier is considered to be a narrow strip of fixed width. Sensors are dispersed randomly on the barrier, and the probability of barrier coverage is studied based on the density of dispersal. It has been shown that when the barrier is sufficiently long, one random dispersal leaves gaps in the coverage, and thus several rounds of dispersal are needed to assure complete barrier coverage [9]. Since random sensor dispersal causes incomplete coverage, in the second approach, several papers assume that sensors, once dispersed, are mobile, and can be instructed to relocate from the initial position to a final position on the barrier in order to achieve complete coverage [6, 7]. Clearly, when a sufficient number of sensors is used, this approach always guarantees complete coverage of the barrier. The problem therefore is assigning final positions to the sensors in order to minimize some aspect of the relocation cost. The variations studied so far include minimizing the *maximum* relocation distance (*MinMax*), the *sum* of relocation distances of sensors (*MinSum*), or minimizing the *number* of sensors that relocate (*MinNum*).

Most of the previous work is set in the one-dimensional setting: the barriers are assumed to be one or more line segments that are part of a line \mathcal{L} , and furthermore, the sensors are initially located on the same line \mathcal{L} . In [6] it was shown that there is an $O(n^2)$ algorithm for the MinMax problem in the case when the sensor ranges are identical. The authors also showed that the problem becomes NP-complete if there are two barriers. A polynomial time algorithm for the MinMax problem is given in [10] for arbitrary sensor ranges for the case of a single barrier, and an improved algorithm is given for the case when all sensor ranges are identical. In [7], it was shown that the *MinSum problem* is NP-complete when arbitrary sensor ranges are allowed, and an $O(n^2)$ algorithm is given when all sensing ranges are the same. Minimizing the number of sensors moved was considered in [11]. Similarly as in the MinSum problem, the MinNum problem is NP-complete when arbitrary sensor ranges are allowed, and an $O(n^2)$ algorithm is given when all sensing ranges are the same.

In this paper we consider the algorithmic complexity of several natural generalizations of the barrier coverage problem with sensors of arbitrary ranges. We generalize the work in [10, 6, 7, 11] in two significant ways. First, we assume that the initial locations of sensors are points in the two-dimensional plane and are not necessarily collinear. This assumption is justified since in many situations a dispersal of sensors on the barrier might not be practical. Second, we consider multiple barriers that are parallel or perpendicular to each other. This generalization is motivated by barrier coverage of the perimeter of an area. We use standard cost measures such as Euclidean or rectilinear distance between initial and final positions of sensors.

1.1 Preliminaries and Notation

Throughout the paper, we assume that we are given a set of sensors $S = \{s_1, s_2, \dots, s_n\}$ located in the plane in positions p_1, p_2, \dots, p_n , where $p_i = (x_i, y_i)$ for some real values x_i, y_i . The sensing ranges of the sensors are r_1, r_2, \dots, r_n , respectively. A sensor s_i can detect any intruder in the closed circular area around x_i of radius r_i . We assume that sensor s_i is mobile and thus can relocate itself from its initial location p_i to another specified location p'_i . A *barrier* b is a closed line segment in the plane. Given a set of barriers $\mathcal{B} = \{b_1, b_2, \dots, b_k\}$, and a set of sensors S in positions p_1, p_2, \dots, p_n in the plane, of sensing ranges r_1, r_2, \dots, r_n , the *barrier coverage problem* is to determine for each s_i its final position p'_i on one of the barriers, so that all barriers are covered by the sensing ranges of the sensors. We call such an assignment of final positions a *covering assignment*. Figure 1 shows an example of a barrier coverage problem and a possible covering assignment. Sometimes we are also interested in optimizing some measure of the movement of sensors involved to achieve coverage. We

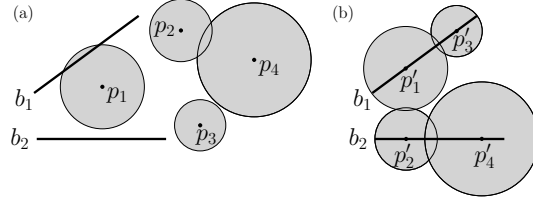


Fig. 1. (a) A given barrier coverage problem (b) a possible covering assignment

are interested in the algorithmic complexity of three problems:

Feasibility problem: Given a set of sensors S located at positions p_1, p_2, \dots, p_n , and a set of barriers \mathcal{B} , determine if there exists a valid covering assignment, i.e. determine whether there exist final positions p'_1, p'_2, \dots, p'_n on the barriers such that all barriers in \mathcal{B} are covered.

MinMax problem: Given a set of sensors S located at positions p_1, p_2, \dots, p_n , and a set of barriers \mathcal{B} , find final positions p'_1, p'_2, \dots, p'_n on the barriers so that all barriers in \mathcal{B} are covered and $\max_{1 \leq i \leq n} \{d(p_i, p'_i)\}$ is minimized.

MinSum problem: Given a set of sensors S located at positions p_1, p_2, \dots, p_n , and a set of barriers \mathcal{B} , find final positions p'_1, p'_2, \dots, p'_n on the barriers so that all barriers in \mathcal{B} are covered, and $\sum_{i=1}^n d(p_i, p'_i)$ is minimized.

1.2 Our Results

Our results are summarized in Table 1. Throughout the paper, we consider the barrier coverage

Barriers	Movement	Feasibility	MinMax	MinSum
1 barrier	Arbitrary final positions	$O(n)$	NP-hard	NP-hard [7]
2 barriers	Arbitrary final positions	NPC	NP-hard	NP-hard
1 barrier	Perpendicular	$O(n)$	$O(n \log n)$	$O(n^2)$
k parallel barriers	Perpendicular	$O(kn)$	$O(kn^{k+1})$	$O(kn^{k+1})$
2 perpendicular barriers	Perpendicular	NPC	NP-hard	NP-hard

Table 1. Barrier coverage problems: Initial positions on the plane, final positions on barriers

problem with sensors of arbitrary ranges, initially located at arbitrary locations in the plane. In Section 2, we assume that sensors can move to arbitrary positions on any of the barriers. While feasibility is trivial in the case of one barrier, it is straightforward to show that it is NP-complete for even two barriers. The NP-hardness of the MinSum problem for one barrier follows trivially from the result in [7]. In this paper, we show that the MinMax problem is NP-hard even for a single barrier. We show that this holds both when the cost measure is Euclidean distance and when it is rectilinear distance.

In light of these hardness results, in the rest of the paper, we consider a more restricted but natural movement. We assume that once a sensor has been ordered to relocate to a particular

barrier, it moves to the closest point on the barrier. We call this *perpendicular movement*⁹. Section 3 considers the case of one barrier and perpendicular movement, while Section 4 considers the case of perpendicular movement and multiple *parallel* barriers. We show that all three of our problems are solvable in polynomial time. Finally, in Section 5, we consider the case of perpendicular movement and two barriers perpendicular to each other. We show that even the feasibility problem is NP-complete in this case. The NP-completeness result holds even in the case when the given positions of the sensors have integer values and the sizes of sensors are limited to two different sizes. In contrast, we give an $O(n^{1.5})$ algorithm for finding a covering assignment for a natural restriction of the problem that includes the case when all sensors are located in integer positions and the sensing ranges of all sensors are of diameter 1.

2 Arbitrary Final Positions

In this section, we assume that sensors are allowed to relocate to any final positions on the barrier(s). We consider standard measures for the cost of relocation, such as Euclidean distance or rectilinear distance.

We consider first the case of a single barrier b . Without loss of generality, we assume that b is located on the x -axis between $(0, 0)$ and $(L, 0)$ for some L . The feasibility of barrier coverage in this case is simply a matter of checking if $\sum_{i=1}^n 2r_i \geq L$. For the MinSum problem, it was shown in [7] that even if the initial positions of sensors are on the line containing the barrier, the problem is NP-hard; therefore the more general version of the problem studied here is clearly NP-hard. Recently, it was shown in [10] that if the initial positions of sensors are on the line containing the barrier, the MinMax problem is solvable in polynomial time. The complexity of the MinMax problem for general initial positions in the plane has not yet been studied and thus we proceed to study the complexity of the MinMax problem when initial positions of sensors can be anywhere on the plane, and the final positions can be anywhere on the barrier. See Figure 2 for an example of the initial placement of sensors.

Theorem 1. *Let $S = \{s_1, s_2, \dots, s_n\}$ be a set of n sensors with ranges r_1, r_2, \dots, r_n located in the plane in initial positions p_1, p_2, \dots, p_n . Let the barrier b be a line segment on the x -axis between $(0, 0)$ and $(L, 0)$. Given an integer k , the problem of determining if there is a covering assignment such that the maximum relocation distance (Euclidean/rectilinear) of the sensors is at most k is NP-complete.*

Proof. Let $R = \sum_{i=1}^n 2r_i$. Clearly if $R < L$ the problem is infeasible, so we assume $R \geq L$. We give here a reduction of the 3-partition problem (see [12]) to the problem of covering the barrier b with sensors such that the maximum movement of the sensors is at most k for the case $R = L$. We are given a multiset $A = \{a_1 \geq a_2 \geq \dots \geq a_n\}$ of $n = 3m$ positive integers such that $B/4 < a_i < B/2$ for $1 \leq i \leq n$ and $\sum_{i=1}^n a_i = mB$ for some B . The problem is to decide whether A can be partitioned into m triples T_1, T_2, \dots, T_m such that the sum of the numbers in each triple is equal to B . We create an instance of the barrier coverage problem as follows: Let $L = mB + m - 1$ so that the barrier b is a line segment from $(0, 0)$ to $(L, 0)$, and let $k = L + 1$. Create a sensor s_i of range a_i for every $1 \leq i \leq 3m$ positioned at $-a_i$. In addition, create $m - 1$ sensors $s_{3m+1}, s_{3m+2}, \dots, s_{4m-1}$ of range $1/2$ located at positions $(B + 1/2, k), (2B + 3/2, k), (3B + 5/2, k), \dots, ((m - 1)B + (2m - 3)/2, k)$. See Figure 2 for an example. Since $R = L$, all sensors must move to the barrier. It is easy to verify

⁹ Note that it is possible for a sensor that is not located on the barrier to cover part of the barrier. However, in this paper, we require final positions of sensors to be on the barrier.

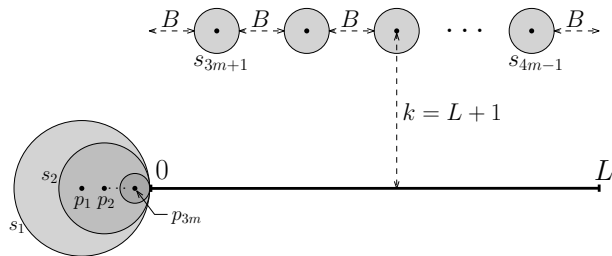


Fig. 2. Reduction from 3-partition to the MinMax problem

that there is a partition of S into m triples T_1, T_2, \dots, T_m , the sum of each triple being B , if and only if there is a solution to the movement of the sensors such that the three sensors corresponding to triple T_i are moved to fill the i th gap in the barrier b and all moves are at most of size k . \square

2.1 Multiple Barriers

It is easy to see that when there are two barriers to be covered, even feasibility of coverage is NP-complete. This can be shown by reducing the Partition problem to an appropriate 2-barrier coverage problem, as in [6]. It follows that k -barrier coverage is also NP-hard.

3 Perpendicular Movement: One Barrier

In this section, we assume that sensors use perpendicular movement, and can only move to the closest point in the barrier. Without loss of generality, let the barrier b be the line segment between $(0, 0)$ and $(L, 0)$ and let the set of n sensors s_1, s_2, \dots, s_n be initially located at positions p_1, p_2, \dots, p_n respectively, where $p_i = (x_i, y_i)$ and $x_1 - r_1 \leq x_2 - r_2 \leq \dots \leq x_n - r_n$. Observe that sensors are ordered by the leftmost x -coordinate they can cover, and they can only move in a vertical direction. For simplicity we assume all points of interest (sensor locations, left and right endpoints of sensor ranges, barrier left and right endpoint) are distinct. Since the y -coordinate of all points on the barrier are the same, we sometimes represent the barrier or a segment of the barrier by an interval of x -coordinates. For technical reasons, we denote the segment of the barrier between the points $(i, 0)$ and $(j, 0)$ by the *half-open interval* $[i, j)$.

We first show a necessary and sufficient condition on the sensors for the barrier to be covered. Since only vertical movements are allowed, given a point $p' = (x', 0)$ on the barrier, a sensor s in position $p = (x, y)$ with sensing range r can be assigned to cover p' only if $|x - x'| \leq r$. Once again, for technical reasons, we consider the interval on the barrier that a sensor can cover to be a half-open interval. More precisely, we say that the sensor s at position $p = (x, y)$ is a *candidate sensor* for $p' = (x', y')$ on the barrier if $x - r \leq x' < x + r$. Alternatively we say s potentially covers the point p' . Clearly, the barrier b can be covered only if every point on the barrier has a candidate sensor. Conversely, if every point has a candidate sensor, the problem can be solved in linear time by simply repeatedly covering the leftmost uncovered point on the barrier by moving the smallest numbered candidate sensor for the point down to the barrier.

3.1 MinSum

We give a dynamic programming formulation for the MinSum problem. We denote the set of sensors $\{s_i, s_{i+1}, \dots, s_n\}$ by S_i . If the barrier is an empty interval, then the cost is 0. If the first sensor is not a candidate for the left endpoint of the barrier, or if the sensor set is empty while the barrier is a non-empty interval, then clearly the problem is infeasible and the cost is infinity. If not, observe that the optimal solution to the MinSum problem either involves moving sensor s_1 down to the barrier or it doesn't. In the first case, the cost of the optimal solution is the sum of y_1 , the cost of moving the first sensor to the barrier, and the optimal cost of the subproblem of covering the interval $[x_1 + r_1, L)$ with the remaining sensors $S_2 = S - \{s_1\}$. In the second case, the optimal solution is the optimal cost of covering the original interval $[0, L)$ with S_2 . The recursive formulation is given below:

$$\text{cost}(S_i, [a, L)) = \begin{cases} 0 & \text{if } L < a \\ \infty & \text{if } x_i - r_i > a \\ \infty & \text{if } S_i = \emptyset \text{ and } L > a \\ \min \begin{cases} y_i + \text{cost}(S_{i+1}, [x_i + r_i, L)), \\ \text{cost}(S_{i+1}, [a, L)) \end{cases} & \text{otherwise} \end{cases}$$

Observe that a subproblem is always defined by a set S_i and a left endpoint to the barrier which is given by the rightmost x -coordinate covered by a sensor. Thus the number of possible subproblems is $O(n^2)$, and it takes constant time to compute $\text{cost}(S_i, [a, L))$ given the solutions to the subproblems. Thus, by using either a tabular method or memoization, the problem can be solved in quadratic time.

Theorem 2. *Let s_1, s_2, \dots, s_n be n sensors initially located at positions p_1, p_2, \dots, p_n respectively, and let b be a barrier between $(0, 0)$ and $(L, 0)$. The MinSum problem using only perpendicular movement can be solved in $O(n^2)$ time.*

The same dynamic programming formulation works for minimizing the maximum movement, except that in the case when the i -th sensor moves down in the optimal solution, the cost is the maximum of y_i and $\text{cost}(S_{i+1}, [x_i + r_i, L))$ instead of their sum. Thus, the MinMax problem can also be solved in $O(n^2)$ time. However, an alternative approach is more efficient. Consider the subset of sensors that are at distance at most d from the barrier. Clearly, we can check the feasibility of covering the barrier with such a subset in $O(n)$ time. The minimum value of d for which the problem remains feasible gives the solution to the minmax problem. This optimal value of d can be found using binary search on the set of distances of all sensors to the barrier. This gives the following result:

Theorem 3. *Let s_1, s_2, \dots, s_n be n sensors initially located at positions p_1, p_2, \dots, p_n respectively, and let b be a barrier between $(0, 0)$ and $(L, 0)$. The MinMax problem using only perpendicular movement can be solved in $O(n \log n)$ time.*

4 Perpendicular Movement: Multiple Parallel Barriers

In this section, we study the problem of covering multiple *parallel* barriers. We assume that sensors can relocate to any of the barriers, but will use perpendicular movement to move to the closest point of the chosen barrier. Without loss of generality, we assume all barriers are parallel to the x -axis. Since there are k barriers, there are k points on barriers with the same x -coordinate. We therefore speak of sensors being *candidates for x -coordinates*: a sensor s in position $p = (x, y)$ with

sensing range r is a candidate sensor for x -coordinate x' if $x - r \leq x' < x + r$. Clearly, such a sensor is a candidate for an point p' on a border with x -coordinate x' . We say an interval $I = [a, b)$ of x -coordinates is k -coverable if every x -coordinate in the interval has k candidate sensors. Observe that such an interval of x -coordinates could exist on multiple barriers.

For simplicity, we explain the case of two barriers; the results for the feasibility and MinSum problems generalize to k barriers. Assume without loss of generality that the two barriers to be covered are b_1 between $(0, 0)$ and $(L, 0)$ and b_2 between $(0, W)$ and (L, W) and the set of n sensors s_1, s_2, \dots, s_n to be initially located at positions p_1, p_2, \dots, p_n respectively, and is ordered by the $x_i - r_i$ values as in Section 3. Thus, sensors may only move in a vertical direction. We assume that the ranges of sensors are smaller than the distance W between the two barriers, and thus it is impossible for a sensor to simultaneously cover two barriers. See Figure 3 for an example of such a problem.

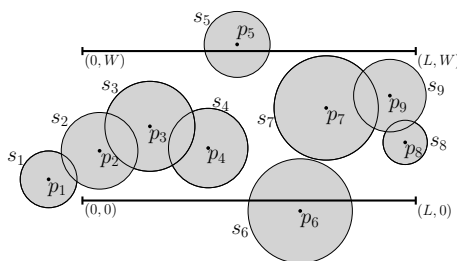


Fig. 3. An example of a barrier coverage problem with two parallel barriers

We first show a necessary and sufficient condition on the sensors for the two barriers to be covered. Clearly, since the ranges of sensors are smaller than the distance between the two barriers, the barrier coverage problem for the two parallel barriers b_1 and b_2 above is solvable by a set of sensors S *only if* the interval $[0, L)$ is 2-coverable by S . We proceed to show that 2-coverability is also a sufficient condition, and give a $O(n)$ algorithm for finding a covering assignment for two parallel barriers. To simplify the proof of the main theorem, we first prove a lemma that considers a slightly more general version of the two parallel barrier problem.

Lemma 1. *Let s_1, s_2, \dots, s_n be n sensors initially located at positions p_1, p_2, \dots, p_n respectively where $p_i = (x_i, y_i)$ and $x_1 - r_1 \leq x_2 - r_2 \leq \dots \leq x_n - r_n$. Let b_1 between $(0, 0)$ and $(L, 0)$ and b_2 between (P, W) and (Q, W) , where $0 \leq P < L \leq Q$, be two parallel barriers to be covered. If intervals $[0, P)$ and $[L, Q)$ are 1-coverable, and interval $[P, L)$ is 2-coverable, then a covering assignment that uses only perpendicular movement of the sensors can be obtained in $O(n)$ time.*

Proof. Omitted due to lack of space. □

The above lemma establishes that complete coverage of two parallel barriers b_1 between $(0, 0)$ and $(L, 0)$ and b_2 between $(0, W)$ and (L, W) can be achieved if and only if the interval of x -coordinates $[0, L)$ is 2-covered, and a covering assignment can be found in linear time. It is easy to see that the lemma can be generalized for k barriers to show that the feasibility problem can be solved in $O(kn)$ time. We proceed to study the problem of minimizing the sum of movements required to perform barrier coverage.

4.1 MinSum

The dynamic programming formulation given in Section 3.1 can be generalized for the case of two barriers. The key difference is that in an optimal solution, sensor s_i may be used to cover a part of barrier b_1 or barrier b_2 or neither. Let $xcost(S_i, [a_1, L], [a_2, L])$ denote the cost of covering the interval $[a_1, L)$ of the barrier b_1 and the interval $[a_2, L)$ of the second barrier with the sensor set $S_i = \{s_i, s_{i+1}, \dots, s_n\}$. The optimal cost is given by the formulation below:

$$xcost(S_i, [a_1, L], [a_2, L]) = \begin{cases} cost(S_i, [a_2, L]) & \text{if } L < a_1 \\ cost(S_i, [a_1, L]) & \text{if } L < a_2 \\ \infty & \text{if } x_i - r_i > \min\{a_1, a_2\} \\ \infty & \text{if } S_i = \emptyset \text{ and } \\ & L > \min\{a_1, a_2\} \\ \min \begin{cases} y_i + xcost(S_{i+1}, [x_i + r_i, L], [a_2, L]), \\ W - y_i + xcost(S_{i+1}, [a_1, L], [x_i + r_i, L]), \\ xcost(S_{i+1}, [a_1, L], [a_2, L]) \end{cases} & \text{otherwise} \end{cases}$$

It is not hard to see that the formulation can be generalized to k barriers; a sensor s_i may move to any of the k barriers with the corresponding cost being added to the solution. Observe that a subproblem is now given by a set S_i , and a left endpoint to each of the barriers. The total number of subproblems is $O(n^{k+1})$ and the time needed to compute the cost of a problem given the costs of the subproblems is $O(k)$. Thus, the time needed to solve the problem is $O(kn^{k+1})$.

Theorem 4. *Let s_1, s_2, \dots, s_n be n sensors initially located at positions p_1, p_2, \dots, p_n respectively where $p_i = (x_i, y_i)$ and $x_1 - r_1 \leq x_2 - r_2 \leq \dots \leq x_n - r_n$. The MinSum problem for k parallel barriers using only perpendicular movement can be solved in $O(kn^{k+1})$ time.*

Clearly a very similar formulation as above can be used to solve the MinMax problem in $O(kn^{k+1})$ time as well. However, the approach used for a single barrier can be used for multiple barriers, as shown in the theorem below:

Theorem 5. *Let s_1, s_2, \dots, s_n be n sensors initially located at positions p_1, p_2, \dots, p_n respectively, and let b_1 between $(0, 0)$ and $(L, 0)$ and b_2 between $(0, W)$ and (L, W) be the two parallel barriers to be covered. The MinMax problem for the 2 parallel barriers using only perpendicular movement can be solved in $O(n \log n)$ time.*

Proof. We first show that given a maximum distance d , we can decide in linear time whether a covering assignment exists so that every sensor relocates at most distance d to its final position. If $d < W/2$, the sets of candidate sensors for each of the two barriers are disjoint. We can verify independently the feasibility of covering each barrier with its set of candidate sensors, as shown in Lemma 1.

If $d \geq W/2$, we partition S into the sets A , B , and C where A consists of sensors that are only candidates for barrier b_1 (that is, they are at distance $> d$ from barrier b_2), B consists of sensors that are only candidates for barrier b_2 , and C consists of candidates for both barriers. We assign all sensors in set A to barrier B_1 and all sensors in set B to barrier B_2 . This now leaves a set of uncovered intervals on each barrier. If there is a point x that is uncovered on either barrier and has no candidate sensors, then barrier coverage is impossible. If there is a point x that is only uncovered on one barrier and has a candidate sensor, then we assign the candidate sensor to the barrier. After

this process is completed, we have a set of intervals that have non-empty parts that are 2-coverable. We now appeal to Lemma 1 to complete the proof.

The optimal value of d can be found using binary search on the set of distances of all sensors from each of the two barriers, and an application of Lemma 1 is done in $O(n)$ time. This completes the proof. \square

5 Perpendicular Movement: Two Perpendicular Barriers

In this section, we consider the problem of covering two perpendicular barriers. Once again, we assume that sensors can relocate to either of the two barriers, but will use perpendicular movement to move to the closest point of the chosen barrier. In contrast to the case of parallel barriers, we show here that even the feasibility problem in this case is NP-complete. Figure 4 illustrates an example of such a problem. For simplicity we assume that b_1 is a segment on the x -axis between $(0, 0), (L_1, 0)$ and b_2 is a segment on the y -axis between $(0, 0), (0, L_2)$. Since the sensors can only employ perpendicular movement, the only possible final positions on the barriers for a sensor s_i in position $p_i = (x_i, y_i)$ are $p'_i = (0, y_i)$ or $p'_i = (x_i, 0)$.

We first show that the feasibility problem for this case is NP-complete by giving a reduction from the monotone 3-SAT problem [12]. Recall that a Boolean 3-CNF formula $f = c_1 \wedge c_2 \wedge \dots \wedge c_m$ of m clauses is called *monotone* if and only if every clause c_i in f either contains only unnegated literals or only negated literals [13]. In order to obtain a reduction into a barrier coverage problem with two perpendicular barriers, we first put a monotone 3-SAT formula in a special form as shown in the lemma below.

Lemma 2. *Let $f = f_1 \wedge f_2$ be a monotone 3-CNF Boolean formula with n clauses where f_1 and f_2 only contain unnegated and negated literals respectively, and every literal appears in at most m clauses. Then f can be transformed into a monotone formula $f' = f'_1 \wedge f'_2$ such that f'_1 and f'_2 have only unnegated and negated literals respectively, and f' has the following properties:*

1. f and f' are equisatisfiable, i.e. f' is satisfiable if and only if f is satisfiable.
2. All clauses are of size two or three.
3. Clauses of size two contain exactly one variable from f and one new variable.
4. Clauses of size three contain only new variables.
5. Each new literal appears exactly once: either in a clause of size two or in a clause of size three.
6. Each x_i appears exactly in m clauses of f'_1 , and exactly in m clauses of f'_2 .
7. f' contains at most $4n + mn$ clauses.
8. The clauses in f'_1 (respectively f'_2) can be ordered so that all clauses containing the literal x_i (\bar{x}_i) appear before clauses containing the literal x_j (respectively \bar{x}_j) for $i < j$, and all clauses of size three are placed last.

Proof. Omitted due to lack of space. \square

We give an example that illustrates the reduction and the ordering specified in Property 7.

Example 1: Consider 3-CNF formula

$$f = (x_1 \vee x_3 \vee x_4) \wedge (x_2 \vee x_3 \vee x_4) \wedge (x_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_4) \wedge (\bar{x}_2 \vee \bar{x}_3 \vee \bar{x}_4)$$

An equisatisfiable formula f' satisfying the properties of Lemma 2 is shown below:

$$\begin{aligned}
f' = & (x_1 \vee x_{1,1}) \wedge (x_1 \vee x_{1,3}) \wedge (x_1 \vee y_{1,1}) \wedge (x_2 \vee x_{2,2}) \wedge (x_2 \vee x_{2,3}) \wedge (x_2 \vee y_{2,1}) \\
& \wedge (x_3 \vee x_{3,1}) \wedge (x_3 \vee x_{3,2}) \wedge (x_3 \vee x_{3,3}) \wedge (x_4 \vee x_{4,1}) \wedge (x_4 \vee x_{4,2}) \wedge (x_4 \vee y_{4,1}) \\
& \wedge (x_{1,4} \vee x_{2,4} \vee x_{4,4}) \wedge (x_{2,5} \vee x_{3,5} \vee x_{4,5}) \\
& \wedge (\overline{x_1} \vee \overline{x_{1,4}}) \wedge (\overline{x_1} \vee \overline{z_{1,1}}) \wedge (\overline{x_1} \vee \overline{z_{1,2}}) \wedge (\overline{x_2} \vee \overline{x_{2,4}}) \wedge (\overline{x_2} \vee \overline{x_{2,5}}) \wedge (\overline{x_2} \vee \overline{z_{2,1}}) \\
& \wedge (\overline{x_3} \vee \overline{x_{3,5}}) \wedge (\overline{x_3} \vee \overline{z_{3,1}}) \wedge (\overline{x_3} \vee \overline{z_{3,2}}) \wedge (\overline{x_4} \vee \overline{x_{4,4}}) \wedge (\overline{x_4} \vee \overline{x_{4,5}}) \wedge (\overline{x_4} \vee \overline{z_{4,1}}) \\
& \wedge (\overline{x_{1,1}} \vee \overline{x_{3,1}} \vee \overline{x_{4,1}}) \wedge (\overline{x_{2,2}} \vee \overline{x_{3,2}} \vee \overline{x_{4,2}}) \wedge (\overline{x_{1,3}} \vee \overline{x_{2,3}} \vee \overline{x_{3,3}})
\end{aligned}$$

Theorem 6. *Let s_1, s_2, \dots, s_n be n sensors initially located at positions p_1, p_2, \dots, p_n respectively, and let b_1 between $(0, 0)$ and $(L_1, 0)$ and b_2 between $(0, 0)$ and $(0, L_2)$ be the two perpendicular barriers to be covered. Then the problem of finding a covering assignment using perpendicular movement for the two barriers is NP-complete.*

Proof. It is easy to see that any given covering assignment can be verified in polynomial time. Given an instance f of monotone 3-SAT we show how to obtain a corresponding instance P of the barrier coverage problem with two perpendicular barriers so that f is satisfiable iff there exist a covering assignment using perpendicular movement for P . Given a monotone 3-SAT formula f , we use the construction described in Lemma 2 to obtain a formula $f' = f'_1 \wedge f'_2$ satisfying the properties stated in Lemma 2 with clauses ordered as described in Property 7. Let f_1 have i_1 clauses, and f_2 have i_2 clauses, and assume the clauses in each are numbered from $1, \dots, i_1$ and $1, \dots, i_2$ respectively. We create an instance P of the barrier coverage problem with two barriers b_1 , the line segment between $(0, 0)$ and $(2i_1, 0)$ and b_2 , the line segment between $(0, 0)$ and $(0, 2i_2)$.

For each variable x_i of the original formula f we have a sensor s_i of sensing range m located in position $p_i = ((2i - 1)m, (2i - 1)m)$, i.e., on the diagonal. Figure 4 illustrates the instance of barrier coverage corresponding to the 3-SAT formula from Example 5 above. Each of the variables $x_{i,j}, y_{i,j}, z_{i,j}$ is represented by a sensor of sensing range 1, denoted $s_{i,j}, s'_{i,j},$ and $s''_{i,j}$ respectively, and is placed in such a manner that the sensors corresponding to variables associated with the same s_i collectively cover the same parts of the two barriers as covered by sensor s_i . Furthermore, sensors corresponding to variables that appear in the same clause of size three cover exactly the same segment of a barrier. A sensor corresponding to a new variable $x_{i,j}$ that occurs in the p th clause in f'_1 and in the q th clause in f'_2 is placed in position $(2p - 1, 2q - 1)$. For example the sensor $s_{1,3}$ corresponding to the variable $x_{1,3}$ appears in the second clause of f'_1 and the fifteenth clause of f'_2 , and hence is placed at position $(3, 29)$. Similarly, the sensor $s_{2,4}$ corresponding to the variable $x_{2,4}$ appears in the thirteenth clause of f'_1 and the fourth clause of f'_2 , and hence is placed at position $(25, 7)$. A sensor corresponding to variable $y_{i,j}$ which occurs in the l th clause in f'_1 is placed in position $(2l - 1, -1)$ and sensor corresponding to variable $z_{i,j}$ which occurs in the l th clause of f'_2 is placed in position $(-1, 2l - 1)$. Observe that in this assignment of positions to sensors, for any i , there is a one-to-one correspondence between the line segments of length 2 in b_1 and b_2 and clauses in f'_1 and f'_2 respectively. In particular, the sensors that potentially cover the line segment from $(2i - 2, 0)$ to $(2i, 0)$ on the barrier b_1 correspond to variables in clause i of f'_1 . Similarly, the sensors that potentially cover the line segment from $(0, 2i - 2)$ to $(0, 2i)$ on the barrier b_2 correspond to variables in clause i of f'_2 .

It is easy to verify that f' is satisfiable iff for the corresponding instance P there exists a covering assignment assuming perpendicular movement. \square

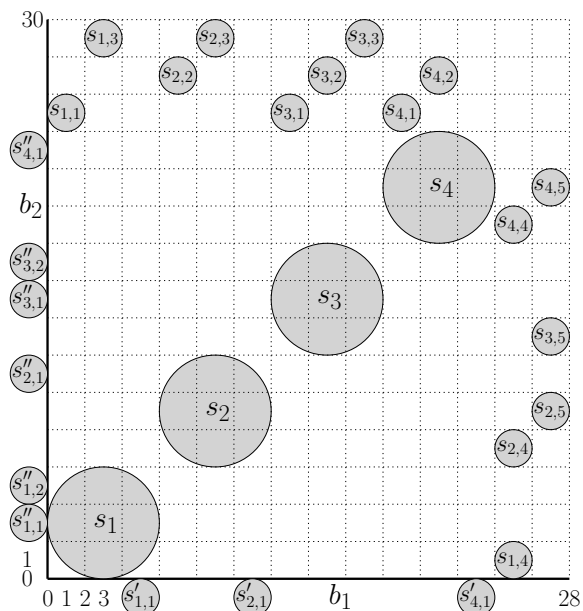


Fig. 4. Barrier coverage instance corresponding to the monotone 3-SAT instance of Example 1

Since any instance of monotone 3-SAT problem can be transformed into one in which no variable occurs more than 4 times, it follows from the proof that the problem is NP-complete even when the sensors are in integer positions and the ranges are limited to two different sizes 1 and $m \geq 4$. It is also clear from the proof that the perpendicularity of the barriers is not critical. The key issue is that the order of intervals covered by the sensors in one barrier has no relationship to those covered in the other barrier. In the case of parallel barriers, this property does not hold. The exact characterization of barriers for which a polytime algorithm is possible remains an open question.

We now turn our attention to restricted versions of barrier coverage of two perpendicular barriers where a polytime algorithm is possible. For S a set of sensors, and barriers b_1, b_2 , we call (S, b_1, b_2) a *non-overlapping arrangement* if for any two sensors $s_i, s_j \in S$, the intervals that are potentially covered by s_i and s_j on the barrier b_1 (and b_2) are either the same or disjoint. This would be the case, for example, if all sensor ranges are of the same diameter equal to 1 and the sensors are in integer positions. We show below that for a non-overlapping arrangement, the problem of finding a covering assignment is polynomial.

Theorem 7. *Let $S = \{s_1, s_2, \dots, s_n\}$ be a set of sensors located in the plane in positions p_1, p_2, \dots, p_n and let b_1 and b_2 be two perpendicular barriers to be covered. If (S, b_1, b_2) form a non-overlapping arrangement, then there exists an $O(n^{1.5})$ algorithm that finds a covering assignment, using only perpendicular movement or reports that none exists.*

Proof. Omitted due to lack of space. □

6 Conclusions

It is known that the problem of minimizing the maximum movement to cover a line segment barrier when the sensors are initially located on the line containing the barrier is solvable in polynomial time [10]. In contrast, our results show that the MinMax barrier coverage problem becomes NP-hard when sensors of arbitrary ranges are initially located in the plane and are allowed to move to any final positions on the barrier. It remains open whether this problem is polynomial in the case when there is a fixed number of possible sensor ranges.

If sensors are restricted to use perpendicular movement, the feasibility, MinMax, and MinSum problems are all polytime solvable for the case of k parallel barriers. However, when the barriers are not parallel, even the feasibility problem is NP-complete, even when sensor ranges are restricted to two sizes. It would be therefore interesting to study approximation algorithms for MinMax and MinSum for this case. Characterizing the problems for which barrier coverage is achievable in polytime remains an open question.

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