Compass routing on geometric networks (1999, Kranakis, Singh, Urrutia) Routing with guaranteed delivery in ad hoc wireless networks (1999, Bose, Morin, Stojmenovic, Urrutia) Routing in geometric networks (2003, Kuhn, Wattenhofer, Zhang, Zollinger)

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1 PROBLEM DEFINITION

Wireless networks are often modelled using geometric graphs. Using only local geometric information to compute a sequence of distributed forwarding decisions that send a message to its destination, routing algorithms can succeed on several common classes of geometric graphs. These graphs' geometric properties provide navigational cues that allow routing to succeed using only limited local information at each node.

1.1 Network Model

A common geometric graph model for wireless networks is to represent each node by a point in the Euclidean plane, $\mathcal{R}^2$, and to add an edge (u, v) for each pair of nodes that can communicate by direct wireless transmission. The absence of the edge (u, v) signifies that u cannot transmit directly to v, requiring a multi-hop transmission via a sequence of intermediate nodes that forms a route from u to v. The cost $c(e)$ of sending a message over an edge $e = (u, v)$ has been modelled in different ways; the most common measures include the *hop (link) metric* ($c(e) = 1$), the *Euclidean metric* ($c(e) = |e|$, where $|e| = \operatorname{dist}(u, v)$ is the Euclidean length of the edge e), and the *energy metric* ($c(e) = |e|^\alpha$ for $\alpha \geq 2$).

In some models, transmission is assumed to be uniform in all directions and of equal range, say r, for all nodes. Under this assumption, the undirected edge (u, v) exists if and only if $\operatorname{dist}(u, v) \leq r$. Thus, for each node v there is an edge from v to every node u that lies within a disk of radius

r centered at v. This is the *unit disk graph* model for wireless networks. Common classes of geometric graphs that are used to model wireless networks include:

Unit Disk Graph. Vertices are points in $\mathcal{R}^2$ and each edge (u, v) exists if and only if $\text{dist}(u, v) \leq r$, for a given fixed $r > 0$.

Plane Graph. Vertices are points in $\mathcal{R}^2$ and no two edges cross.

Triangulation. Vertices are points in $\mathcal{R}^2$ and every interior face is a triangle.

Quasi-unit Disk Graph. Vertices are points in $\mathcal{R}^2$ and each edge (u, v) exists if $\text{dist}(u, v) \leq r_1$, may exist if $r_1 < \text{dist}(u, v) \leq r_2$, but does not exist if $\text{dist}(u, v) > r_2$, for given fixed $r_2 > r_1 > 0$.

Unit Ball Graph. Vertices are points in $\mathcal{R}^3$ and each edge (u, v) exists if and only if $\text{dist}(u, v) \leq r$, for a given fixed $r > 0$.

Gabriel Graph. Vertices are points in $\mathcal{R}^2$ and each edge (u, v) exists if and only if the disk with diameter (u, v) does not contain any other vertices.

Other classes of geometric graphs used to model wireless networks include relative neighbourhood graphs, Delaunay triangulations, Yao graphs, convex subdivisions, monotone subdivisions, edge-augmented plane graphs, and physically-based models such as SINR.

A geometric graph G is *civilized* with λ*-precision* if for every pair of nodes u and v in G, $\text{dist}(u, v) \geq \lambda$ for a given fixed $\lambda > 0$, where λ is independent of n, the number of nodes in G.

1.2 Communication Protocol

In several wireless network protocols, e.g., ad hoc or wireless sensor networks, there is no fixed infrastructure for routing nor any central servers. All nodes act as hosts as well as routers. Apart from a node's immediate neighbourhood, the topology of the network is unknown, i.e., each node is aware of its own location (its (x, y) coordinates) as well as the coordinates of its neighbours. Nodes must discover and maintain routes in a distributed manner without knowledge of precomputed routing tables, any particular vertex labelling (other than spatial coordinates), nor the support of a central server. Additionally, some models incorporate constraints for limited memory and power. Depending on the particular model, a limited amount of information can be stored in message headers to assist with routing. When a node receives a message, it reads the header (possibly modifying the header information) before selecting one of its neighbours to which to forward the message. A *stateless* algorithm does not modify the header. Networks nodes have no memory themselves; any dynamic state information is stored in the message header. Furthermore, no precomputed information about the network is known to the nodes.

1.3 Geometric Routing

Given the coordinates of a target node t in a (wireless) geometric network G, a source node s in G is tasked with sending a message via a multi-hop route through G from s to t. Routing proceeds by computing a sequence of distributed forwarding decisions, where each node along the route selects one of its neighbours to which to forward the message. Geometric routing is uniform in that all nodes execute the same protocol. Each node makes a forwarding decision as a function of its coordinates, the coordinates of its neighbours, the coordinates of t, and any available state bits stored in the message header. The number of state bits available is critical to guaranteeing delivery in some classes of geometric graphs by enabling the route to avoid looping and reach t. A node may modify the state bits before forwarding the message. In some models, this state information corresponds to storing data about $O(1)$ nodes, e.g., storing the coordinates of $O(1)$ nodes.

The primary objective is to guarantee message delivery to the target node t. Secondary objectives include minimizing the total cost of communication (the sum of $c(e)$ for all edges e on the route) and minimizing the worst-case or average *dilation* (the ratio of the cost of the route followed relative to that of the route of lowest cost). These secondary objectives are motivated by the need for nodes to conserve power in many wireless networking settings.

2 KEY RESULTS

Local geometric routing assumes only limited control information stored in message headers and local information available at each node along the route. This locality provides network independence that results in natural scalability to larger networks and continued functionality after arbitrary changes to the network. A routing algorithm is said to *succeed* on a particular class of geometric graphs if it guarantees delivery from any source node s to any target node t on any graph in the class; otherwise, the algorithm *fails* on that class of graphs.

Below we summarize key local geometric routing algorithms and their properties.

Greedy Routing. Upon receiving a message, a node forwards it to its neighbour closest to the target node t. Greedy routing is stateless. This strategy succeeds on Delaunay triangulations, but fails on more general classes of geometric graphs such as non-Delaunay triangulations, convex subdivisions, plane graphs, and unit disk graphs.

Compass Routing [7]. Upon receiving a message, a node u forwards it to its neighbour v that minimizes the angle $\angle vut$ with the target node t. Compass routing is stateless. This strategy succeeds on regular triangulations, but fails on more general classes of geometric graphs such as non-regular triangulations, convex subdivisions, plane graphs, and unit disk graphs.

Greedy-Compass Routing [1]. Upon receiving a message, a node u considers its two neighbours on either side of the line segment $\overline{ut}$ (node u's *compass neighbours*) and forwards the message to the one closest to t. Greedy-compass routing is stateless. This strategy succeeds on all triangulations, but fails on more general classes of geometric graphs such as convex subdivisions, plane graphs, and unit disk graphs.

Bose et al. [1] show that no stateless algorithm can succeed on convex subdivisions (including plane graphs and unit disk graphs). Therefore, to succeed on classes of geometric graphs beyond triangulations, local routing algorithms require storing one or more state bits in the message header or predecessor information, i.e., the coordinates of the node that last forwarded the message.

One State Bit [3]. Upon receiving a message, a node u chooses between forwarding the message to its clockwise or counter-clockwise compass neighbour, depending on the value of a state bit. If the compass neighbour lies opposite the vertical line through t, the state bit is flipped. This algorithm uses a single state bit. This strategy succeeds on all triangulations and convex subdivisions, but fails on more general classes of geometric graphs such as plane graphs and unit disk graphs.

Predecessor Awareness and Monotonicity [3]. Each node locally identifies its topmost left neighbour as its parent and its right neighbours as its children. With knowledge of the predecessor, the node forwards the message to its $(i + 1)$st child after receiving it from its ith child, and eventually back to its parent after receiving it from its last child. The resulting route contains a depth-first traversal of a spanning tree of the network. This algorithm is stateless, but each node requires knowledge of its predecessor, i.e., the coordinates of the node that last forwarded the message. This strategy succeeds on triangulations, convex subdivisions, monotone subdivisions, and edge-augmented graphs from these classes, but fails on more general classes of geometric graphs such as non-monotone plane graphs and unit disk graphs.

Face Routing [4, 7]. The message is forwarded along the perimeters of faces in the sequence of faces that intersect the line segment from the source node s to the target node t. This strategy applies the *right-hand principle*, in which each face in the sequence is traversed in a counter-clockwise direction, as if one were walking while sliding the right hand along the wall. To avoid cycling indefinitely, the algorithm must store the coordinates of $O(1)$ nodes that act as progress markers. Furthermore, each node requires knowledge of its predecessor. This strategy succeeds on plane graphs, including triangulations, convex subdivisions, and Gabriel graphs. The intersection of a unit disk graph with the Gabriel graph of a set of points is planar and remains connected if the original unit disk graph is connected. Furthermore, this subgraph can be computed locally; this property allows face routing to succeed on unit disk graphs [4], as well as quasi-unit disk graphs with bounded ratio $r_2/r_1 < \sqrt{2}$ and unit ball graphs contained within slabs of thickness less than $1/\sqrt{2}$ [6]. Although unit disk graphs are non-planar in general, the non-planarity is localized; face routing fails on more general classes of non-planar geometric graphs such as quasi-unit disk graphs and unit ball graphs [6], and edge-augmented plane graphs. Face routing can have dilation $\Theta(n)$, where n is the number of network nodes.

Adaptive Face Routing (AFR) [9]. Adaptive face routing is a variant of face routing that achieves optimality on civilized unit disk graphs and civilized planar graphs with the Gabriel property. Like face routing, $O(1)$ state data are stored in the message header and each node requires knowledge of its predecessor. The algorithm attempts to estimate the length c of the shortest path from s and t by $\widehat{c}$ (starting with $\widehat{c} = 2|\overline{st}|$ and doubling it in every consecutive round). In each round, the face traversal is restricted to the region formed by the ellipse with the major axis $\widehat{c}$ centered on $\overline{st}$. Each edge is traversed at most four times, and the dilation achieved is $\Theta(c)$.

Geometric Ad-hoc Routing (GOAFR$^+$) [8]. Combining methods from greedy routing, face routing, and adaptive face routing allows this hybrid algorithm to meet the bounds of adaptive routing on any unit disk graphs and planar graphs with the Gabriel property (not necessarily civilized). The algorithm first applies greedy routing and switches to face routing when the routed message enters a local minimum (a dead end), before again resuming greedy routing as early as possible by applying an *early fallback* technique.

2.1 General (Non-Geometric) Networks

Is geometry necessary for local routing to succeed? Even with knowledge of the predecessor, stateless routing algorithms require knowledge of the induced subgraph of nodes up to distance $n/3$ away in the worst case [2]. That is, stateless routing using only *local* information is impossible. With $\Theta(\log n)$ state bits, local routing on arbitrary (not necessarily geometric) graphs is possible by deterministically recomputing a polynomial-length universal traversal sequence at each node along the route, where $\Theta(\log n)$ bits store an index into the sequence [5].

3 OPEN PROBLEMS

If a node's coordinates can be stored using $O(\log n)$ bits (e.g., if network nodes are positioned on a $n^c \times n^c$ grid), then face routing can be applied using $O(\log n)$ state bits. It remains open whether any local geometric routing algorithm can succeed on plane graphs using $o(\log n)$ state bits. Similarly, it would be interesting to characterize broad classes of geometric graphs on which local geometric routing is possible using $O(1)$ state bits. In addition to guaranteeing delivery, bounding dilation is of interest. E.g., can $O(1)$ dilation be guaranteed on convex subdivisions using $O(1)$ state bits? Finally, the problem of traversing a graph (visiting all nodes) by a sequence of local forwarding decisions is interesting. Stateless algorithms are impossible for any non-Hamiltonian network. How many state bits are necessary for a local algorithm to traverse a triangulation?

4 CROSS REFERENCES

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