Reconstructing Polygons from Scanner Data

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1 Introduction

Range scanners are used in many configurations: looking in to capture objects on a platform or in-situ, looking down to capture terrain or urban environments, or looking out to capture rooms or factory floors. In addition to point coordinates, different scanners may be able to provide surface labels, normals, or unobstructed segments of scanned rays.

The problem of reconstructing a surface from a set of data points has been studied for both theoretical and practical interests. Theoretical solutions can provably reconstruct the original surface when the samples are sufficiently dense relative to local feature size. Applied solutions handle noisy data and often incorporate additional information along with point coordinates, such as estimated normals [3].

We consider problems of reconstructing the 2D floor plan of a room from different types of scanned data – specifically whether knowledge about the geometry (monotonicity, orthogonality) or topology (connectedness, genus) of the room allows efficient reconstruction from less dense data.

1.1 Models and problem definition

We consider five models for input scanner data: A *point scan* is a set of points on walls. A *point-wall scan* is a set of points on walls including the line containing each wall. A *point-normal scan* is a set of points with normals perpendicular to the corresponding walls, each towards the room's interior. A *segment scan* is a point-wall scan for which each point records the position of its scanner; the line segment from scanner to point must be inside the room. A *visibility-polygon scan* is a set of visibility polygons, i.e., the entire region visible from each scanner. Let n denote number of elements in a scan.

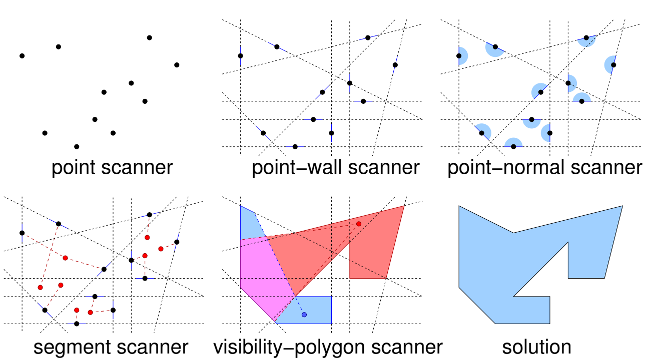

Figure 1: Instances of the five models and a solution.

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We ask whether there exists a polygon that is consistent with the scanned data under the assumption that all walls have been seen, i.e., that each polygon edge contains at least one scanned point. When scanning from a single position, only star-shaped polygons can be reconstructed. We may impose restrictions on topology or geometry: that the room is simply connected and that the walls are orthogonal (every edge is parallel to a coordinate axis) or monotone (any vertical line intersects the boundary in at most two segments).

1.2 Related work

The problem has been well studied if the scanned points coincide with polygon vertices, rather than points on edge interiors. When edges must meet a vertex at right angles, O'Rourke [4] gives an $O(n \log n)$ time algorithm to construct a solution polygon, which is unique if it exists. The problem is NP-hard if both straight and right angles are permitted [5] or if edges must be parallel to one of three (or more) given directions [1].

1.3 Our results

For a point scan, a solution polygon always exists and can be computed in $O(n \log n)$ time, even if we require it to be star-shaped or monotone. An orthogonal polygon solution does not always exist. Details are omitted.

For scans with more information, the problem is NP-hard (Section 4), even for orthogonal polygons. Some special cases can be solved in polynomial time: orthogonal monotone polygons for point-wall scans (Section 2), monotone polygons for point-normal scans (Section 3), and star-shaped polygons for point-normal scans (details are omitted).

Finally, for most combinations of output restrictions we know whether a solution to each model is unique; we omit these results due to space constraints.

2 Orthogonal Monotone Polygons

In this section we consider the reconstruction problem for orthogonal monotone polygons from point-wall scans. Each input point corresponds either to a horizontal (*H*) or vertical (*V*) edge. The points can be represented by a sequence σ of *V*s and *H*s corresponding to their left-to-right ordering.

Theorem 1 *A monotone orthogonal polygon can be reconstructed from a point-wall scan in $O(n \log n)$ time; the solution is unique.*

Proof: (Sketch) If a solution exists, σ contains equally many *V*s and *H*s, and begins and ends with *V*. Consequently, σ contains *HH* as substring; we reconstruct the polygon starting from this point. For two consecutive horizontal edges, the one with greater y-coordinate must be in the upper chain and the other in the lower chain. If the next two elements of σ are *VH*, then the relative positions of these two data points tell us to which chain the vertical edge belongs. The horizontal edge then belongs to the same chain as the vertical edge. Following this step, we again know the y-coordinate of both the upper and lower chains, so if the next two letters of σ are again *VH*, we can resolve these two edges, and so on.

If the next substring is not *VH*, then it must be *H* (which can easily be handled) or *VV* (we can determine to which chains the vertical edges belong, but then cannot move on). The substring *HH* must follow (possibly later in the sequence) each occurrence of a *VV* substring. We jump forward to this occurrence of *HH* resolve rightward from that point on until we reach another *VV*, jump forward to the next *HH*, and so on, until we reach the rightmost data point.

We then repeat the same procedure in the opposite direction: start at the rightmost *HH*, resolve each substring *HV* leftward (in a symmetric manner) until we reach *VV*, jump leftward to the next *HH*, and so on, until we reach the leftmost data point. Two consecutive *VV* substrings must have an *HH* between them since σ corresponds to an orthogonal monotone polygon. This resolves the complete polygon and the solution is unique. □

3 Monotone Polygons

In this section we consider the reconstruction problem for monotone polygons from a point-normal scan. Each input point knows the orientation and interior of the polygon boundary passing through it. In the monotone setting, these half-spaces determine whether each edge belongs to the upper or lower chain of the polygon boundary (with the exception of vertical edges).

Theorem 2 *A monotone polygon can be reconstructed from a point-normal scan in* $O(n \log n)$ *time.*

Proof: (Sketch) We use dynamic programming to assign vertical edges to the top or bottom chains. Scan the data points from left to right and update a function that stores whether there is a partial solution (upper and lower chains) up to the current x-coordinate t. If the vertical line through t contains no data point, then the upper chain must be on the line through the last data point in the upper chain before t, or through the next data point in the upper chain after t. Similarly for the lower chain. This gives four combinations for which the upper and lower chains end. We store whether a partial solution exists for each of them, and update accordingly whenever any two of these four lines cross, or whenever we reach the x-coordinate of the next data point. □

4 Hardness Results

Given an orthogonal graph G (a graph embedded in the plane with edges drawn as axis-parallel line segments), it is NP-hard to determine whether G has a crossing-free spanning tree [2]. This problem is the basis for our reduction to show:

Theorem 3 *Polygon reconstruction under the visibility-polygon scan model is NP-hard.*

Proof: (Sketch) Given any orthogonal graph G, we construct an instance of the visibility-polygon scan problem by replacing each vertex v with the vertex gadget illustrated in Fig. 2. In this gadget, there is a gap in the corresponding polygon edge for every neighbour of v. This allows either connecting to the neighbouring vertex gadget via a corridor formed by a pair of parallel edges (blue), or closing the gap by extending an edge (red).

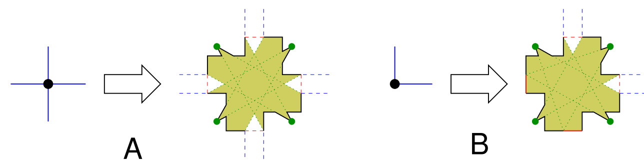

Figure 2: Vertex gadgets with degree 4 (A) and degree 2 (B). Dashes indicate how an edge may be closed or continued, provided it matches a neighbouring gadget.

If G has a non-crossing spanning tree, then our instance has a simple polygonal solution formed by including the corridors that correspond to edges of the spanning tree. The reverse direction can also be shown to hold. □

It is straightforward to modify the vertex gadget such that all edges are orthogonal, showing that the visibility-polygon scan problem remains NP-hard under orthogonality.

5 Discussion and Directions for Future Research

If a solution is not unique, we may ask how many additional scanners are necessary to reveal the true solution. This question is NP-hard since Theorem 3 shows hardness for an instance of the the corresponding decision problem. The problems of reconstructing a monotone polygon or a star-shaped polygon from point-wall scanners remain open, as do the corresponding problems in higher dimensions.

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