Mobile Facility Location

MITACS

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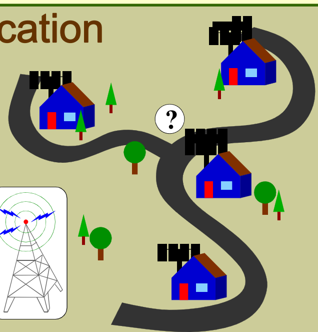
•The problems of ***static*** facility location have been examined for several decades. Only within the last few years have these questions been posed in a ***mobile*** setting.
•Given a set of clients ***moving continuously*** over time, a new set of problems is discovered. These include ***bounding velocity***, maintaining ***continuity***, and ***approximating*** the location of a mobile facility.
•The techniques employed to solve a particular facility location problem do not necessarily extend to a solution to its mobile counterpart.
•The challenges presented by mobile facility location find themselves particularly relevant given the applicability of mobile computing to the wireless telecommunication industries.

1. Static Facility Location

INPUT
Given a set of client positions, $P = \{p_1, \ldots, p_n\} \subseteq \Re^2$,

OUTPUT
identify a position for a facility, $f \in \Re^2$.

The most common problems in facility location are the ***centre*** and the ***median***.

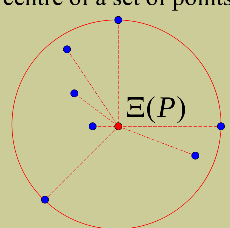


2. Centre

The ***Euclidean centre***, or centre of the smallest enclosing circle, provides a natural definition for the centre of a set of points.

$\Xi(P)$ is the unique point that minimizes

$$\max_{p \in P} \|\Xi(p) - p\|.$$

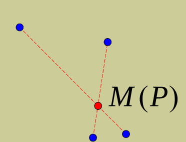


3. Median

Whereas the Euclidean centre minimizes the ***maximum*** distance to the points of P, the ***Euclidean median***, or Weber point, minimizes the ***sum*** of the distances to the points of P.

When the points of P are not collinear, $M(P)$ is the unique point that minimizes

$$\sum_{p \in P} \|M(p) - p\|.$$



4. Mobile Facility Location

INPUT
Given a time interval, $T = [t_0, t_f]$,
and a set of client position functions, $P = \{p_1, \ldots, p_n\}$,
where, $p_i = T \rightarrow \Re^2$,

OUTPUT
identify a position function for a facility, $f = T \rightarrow \Re^2$.

5. Velocity

The velocity of p_i is bounded by v if

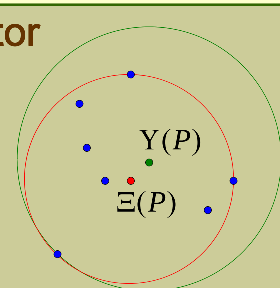
$$\forall t_1, t_2, \quad \|p_i(t_1) - p_i(t_2)\| \le v|t_1 - t_2|.$$

Even when the velocity of clients is bounded, the Euclidean centre and Euclidean median move with unbounded velocity.

Consequently, defining a mobile facility that moves with bounded velocity (relative to the clients' velocities) requires approximation.

6. Approximation Factor

How well does a function approximate the Euclidean centre or the Euclidean median?



Υ is a λ-approximation if

$$\max_{p \in P} \|\Upsilon(P) - p\| \le \lambda \max_{p \in P} \|\Xi(P) - p\|. \quad (\Upsilon \text{ is a centre function})$$

or $$\sum_{p \in P} \|\Upsilon(P) - p\| \le \lambda \sum_{p \in P} \|M(P) - p\|. \quad (\Upsilon \text{ is a median function})$$

7. Goals

A mobile facility must balance two opposing goals:

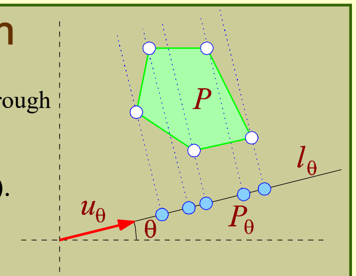
1. maintain a low upper bound on ***maximum velocity***, and
2. provide a good ***approximation factor***.

8. Our Solution

We apply the ***Steiner centre***, originally defined on polytopes, as a centre function within the context of mobile facility location. We introduce a new median function, the ***projection median***. Both provide good approximation factors and low upper bounds on their maximum velocities.

9. Projection

Let l_θ be the line through the origin parallel to unit vector $u_\theta = (\cos\theta, \sin\theta)$.



Let P_θ denote the projection of P onto l_θ.

$$P_\theta = \{u_\theta \langle p, u_\theta \rangle \mid p \in P\}.$$

10. Steiner Centre & Projection Median

The ***Steiner centre*** is defined by integrating over the ***midpoints*** of all P_θ,

$$\Lambda(P) = \frac{2}{\pi} \int_0^\pi \text{mid}(P_\theta)\, d\theta.$$

The ***projection median*** is defined by integrating over the ***medians*** of all P_θ,

$$\Pi(P) = \frac{2}{\pi} \int_0^\pi \text{med}(P_\theta)\, d\theta.$$

11. Evaluation

Both the mobile Steiner centre and the mobile projection median have a maximum velocity of $4/\pi$ relative to the velocity of clients.

The Steiner centre guarantees an approximation of the Euclidean centre to within a factor of 1.1153.

The projection median guarantees an approximation of the Euclidean median to within a factor of $4/\pi$.

These compare very well against other common approximation functions.