Approximating the Independent Domatic Partition Problem in Random Geometric Graphs – An Experimental Study

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Abstract

We investigate experimentally the Domatic Partition (DP) problem, the Independent Domatic Partition (IDP) problem and the Idomatic partition problem in Random Geometric Graphs (RGGs). In particular, we model these problems as Integer Linear Programs (ILPs), solve them optimally, and show on a large set of samples that RGGs are independent domatically full most likely (over 93% of the cases) and domatically full almost certainly (100% of the cases). We empirically confirm using two methods that RGGs are not idomatic on any of the samples. We compare the results of the ILP-based exact algorithms with those of known coloring algorithms both centralized and distributed. Coloring algorithms achieve a competitive performance ratio in solving the IDP problem [12, 11]. Our results on the high likelihood of the "independent domatic fullness" of RGGs lead us to believe that coloring algorithms can be specifically enhanced to achieve a better performance ratio on the IDP size than [12, 11]. We also investigate experimentally the extremal sizes of individual dominating and independent sets of the partitions.

1 Introduction and Motivation

The domatic partition (DP) problem is a classical problem in graph theory whose goal is to partition a graph G into disjoint dominating sets. The *domatic number* $d(G)$ is the maximum number of dominating sets in such a partition [4]. The concept has various applications such as the strategic placement of objects on the nodes of a network (facility location) [18, 3] or the identification of the maximum number of disjoint transmitting groups in a network [7]. Recently, the DP problem found applications for efficient replica placement in Peer-to-Peer systems and for cooperative caching between ISPs to improve video on-demand delivery strategies [19, 1]. More relevant to our work is the application to energy conservation and sleep scheduling in Wireless Sensor Networks (WSN) [17, 16, 8, 10, 12, 11] which are often modeled in practice as Random Geometric graphs (RGGs). A random geometric graph $G(n, r)$ is defined by n vertices uniform in the unit square with an edge between any two vertices of V within Euclidean distance r of each other. An RGG simply induces a uniform probability distribution on a Unit Disk Graph (UDG). A variation of the DP problem is the Independent Domatic Partition (IDP) problem which seeks to partition a graph G into disjoint independent dominating sets. The *independent domatic number* $d_{ind}(G)$ is the maximum size of such a partition.

For any graph G, $d_{ind}(G) \leq d(G) \leq \delta(G) + 1$ where $\delta(G)$ denotes the minimum degree in G. If $d(G) = \delta(G) + 1$ and/or $d_{ind}(G) = \delta(G) + 1$, then G is called *domatically full* and/or *independent domatically full* respectively [4]. A graph whose vertices V can be strictly partitioned into disjoint independent dominating sets is termed *indominable* [2] or *idomatic* [4]. The *idomatic number* $id(G)$ is the partition's maximum size.

The study described herein is motivated by the desire to empirically verify the existence of the upper bound of $\delta + 1$ disjoint independent dominating sets in RGGs (which model Wireless Sensor Networks). Namely, are random geometric graphs *independent domatically full* in practice?

A $k-coloring$ of a graph $G = (V, E)$ is a partition $\Pi = V_1, V_2, ..., V_k$ of the vertex set $V(G)$ into independent sets V_i, each of which is called a *color class*. A vertex $v \in V_i$ is called $colorful$ if each color $1 \leq i \leq k$ appears on the closed neighborhood of v. A k-coloring f is called a $fall$ k-coloring if every vertex in f is colorful. Clearly, a strict partition of the vertex set $V(G)$ into k independent dominating sets (idomatic partition) is equivalent to finding a $fall$ k-coloring of G [9]. We note that if $V_1, V_2, ..., V_{\delta+1}$ are disjoint independent dominating sets of G, then the induced subgraph $\bigcup_{i=1}^{\delta+1} V_i$ is a maximal idomatic subgraph of G of the same minimum degree.

Moreover, we experimentally study the "domination chain" $\gamma(G) \leq i(G) \leq \beta_0(G)$ in RGGs. The "domination chain" is a relation between graph parameters that is satisfied in any graph G [4], where $\gamma(G)$ is the size of the minimum dominating set (MDS) termed the *domination number*, $i(G)$ is the size of the minimum independent dominating set (MIDS) termed the *independence domination number* and $\beta_0(G)$ is the size of the maxi-

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mum independent set (MaxIS) termed the *independence number*. Finding these values are NP-complete problems in general graphs and Unit Disk Graphs [4, 13].

The decision version of the DP problem for $k \geq 3$ is NP-complete in general graphs, circular arc graphs, split graphs and bipartite graphs. It is in P for interval graphs, proper circular arc graphs and strongly chordal graphs [3, 7]. It is an open question whether the problem is also NP-complete in Unit Disk Graphs, but it most likely is [14]. The IDP and Idomatic partition (fall coloring) problems are NP-complete in general graphs and k-regular and bipartite graphs as well [5, 9]. We know of two constant factor approximation algorithms to the domatic partition [17] and connected domatic partition [16] problems in Unit Disk Graphs (UDG). In [12, 11] we show competitive performance ratios by using graph coloring algorithms to empirically approximate the DP and IDP problems.

2 Our Contributions

In this paper, our main contributions are:

-We solve the IDP problem optimally and show that over 93% of the RGG instances are independent domatically full and 100% of the instances are domatically full. The high likelihood of the existence of an optimal partition of $\delta + 1$ independent dominating sets in typical RGGs suggests that coloring algorithms can be fine-tuned to achieve a better performance ratio [12, 11].

-We confirm by Smallest Last (SL) coloring [15] for a large sample of RGG instances that $\chi(G) \geq \omega(G) > \delta(G) + 1$, hence these graphs cannot be *idomatic* [2]. In addition, we formulate the idomatic partition problem as an ILP and confirm through experiments that all graphs of the sample are not *idomatic*.

-We experimentally study the node packing in the sets of the IDP solution and also report on the domination chain values and compare the results obtained by ILP algorithms and coloring algorithms with the asymptotic bounds based on "optimal" triangular lattice packing.

We believe this study answers relevant questions for practitioners and also stimulates further research on the approximability of the IDP problem in UDGs and RGGs and on the asymptotic behavior of domination and domatic properties in RGGs.

3 Algorithms

IDP Formulation. Given a graph $G = (V, E)$ and the set $K = \{1, ..., \delta + 1\}$, we formulate the IDP problem as the following Integer Linear Program (ILP):

$$\text{maximize} \sum_{k=1}^{\delta+1} u_k$$

$$\text{s.t. } x_u^k + \sum_{v:(u,v)\in E} x_v^k \geq u_k \ \forall u \in V, k \in K \quad (1)$$

$$x_v^k + x_u^k \leq 1 \ \forall u, v \in V : (u, v) \in E, k \in K \quad (2)$$

$$\sum_{k=1}^{\delta+1} x_u^k \leq 1 \ \forall u \in V \quad (3)$$

$$u_k \in \{0, 1\}, x_u^k \in \{0, 1\} \ \forall u \in V, k \in K \quad (4)$$

where u_k=1 if dominating set $S_k = \{u | x_u^k = 1\}$ is selected in the IDP and u_k=0 otherwise. Constraint (1) expresses domination, (2) independence, (3) node disjointness, i.e. a node can be part of at most one set, and (4) variable integrality. We also formulate the idomatic partition problem as an ILP where we maximize the size of the independent domatic partition as well as the total number of packed nodes in the sets of the partition. The exact algorithms for the MDS, MIDS and MaxIS problems are also modeled as ILPs. For illustrative purposes, we present the ILP formulation of MIDS below:

$$\text{minimize} \sum_{u\in V} x_u$$

$$\text{s.t. } x_u + \sum_{v:(u,v)\in E} x_v \geq 1 \ \forall u \in V \quad (1)$$

$$x_u + x_v \leq 1 \ \forall u, v \in V : (u, v) \in E \quad (2)$$

$$x_u \in \{0, 1\} \ \forall u \in V \quad (3)$$

Coloring Heuristics. In this study, in order to experimentally approximate the IDP problem, we use 5 centralized graph coloring heuristics: Smallest Last, Largest First, Lexicographic, Radial Sweep and Random. These algorithms are described in detail in [12]. We also experiment with 4 distributed coloring heuristics: Trivial Greedy, Largest First, Lexicographic, and 3Cliques-Last. We discuss these algorithms with ample detail in [11] .

4 Experimental Results

In this paper, ILP models are solved optimally using CPLEX 10.0 installed on a Dual Quad Core Intel Xeon X5570 with 72 GB RAM running CentOS Linux 2.6.18. Each core is clocked at 3.00 GHz. The coloring algorithms are implemented in *C#.Net* (Microsoft Visual Studio 2005) on an Intel Core 2 Duo E8400 processor clocked at 3.00 GHz with 3 GB RAM running Windows Vista Enterprise SP1. Our data set consists of 15 graphs generated randomly with δ ∈{5,10,20} and n ∈{50,100,200,400,800}. Results for each (δ, n) pair are averaged over 20 RGG instances, except for the $(\delta = 20, n = 800)$ case where we average over 10 instances, given that the running times of the ILP models were prohibitively long. This provides a sample of 290 test RGG instances that we choose all to be connected. An RGG instance of parameters (δ, n) is selected as follows: First, we generate all n vertices' (x,y) coordinates i.u.d in the unit square then we sort in non-decreasing order all possible $n(n-1)/2$ edges by their Euclidean distance. Following an evolutionary random graph generation paradigm [6], we add the edges to the graph one-by-one in increasing length until the minimum degree over all n vertices equals δ. The edge length that achieves the desired δ represents r of the graph $G(n, r)$. The values of δ are picked to be representative of WSNs modeled as RGGs where typical node degrees cannot be too high. The exact ILP-based algorithms have a running time that can be exponential in the size of the input, whereas the coloring heuristics achieve a competitive performance ratio on RGGs in polynomial time.

4.1 Domination and Independence in RGGs

Table 1 reports the exact values of the domination chain parameters $\gamma(G)$, $i(G)$ and $\beta_0(G)$ by solving the ILP models of the MDS, MIDS and MaxIS problems. For indicative purposes, we report the average radius $\overline{r}$ calculated over the set of 20 r values selected to achieve the desired δ for each one of the 20 RGG instances representative of a given (δ, n) pair. Based on a triangular lattice packing argument, we showed in [10, 12] lower and upper bounds on the size of a maximal (dominating) independent set, which we denote respectively as i^{tr} and β_0^{tr}. Namely, $i^{tr} = \frac{1}{3}.[1/(r^2\frac{\sqrt{3}}{2})]$ and $\beta_0^{tr} = 1/(r^2\frac{\sqrt{3}}{2})$. We also use $\beta_n(r) = (1 + 1/r)^2$ as the absolute upper bound on the size of a maximum independent set in a random geometric graph $G(n, r)$ [10].

We observe that $i^{tr} \leq \gamma(G) \leq i(G) \leq \beta_0(G) \leq \beta_0^{tr} \leq \beta_n(r)$. However, in certain cases, e.g. $(\delta = 10, n = 50)$, we have $\beta_0(G) > \beta_0^{tr}$. In other words, the computed exact value of the *independence number* $\beta_0(G)$ is greater than the expected triangular lattice-based upper bound β_0^{tr}. We attribute this to a boundary effect in the unit square which produces a value of β_0^{tr} smaller than if we had an infinite unbounded lattice. Furthermore, we report that in 288 cases out of 290 (99.3%), $\gamma(G) = i(G)$. By Theorem [4], if G is a graph containing no induced subgraph isomorphic to $K_{1,3}$ (i.e. G is claw-free), then $\gamma(G) = i(G)$. We verified, however, that all graphs have, in fact, at least one claw. This is simply an empirical verification that the theorem is a conditional but not a biconditional.

Table 2 shows the extremal sizes of individual independent dominating sets obtained in the IDP partitions. For lack of space, we only show the results of the ILP exact model and those of two greedy coloring heuristics: Smallest Last (SL), a centralized topology-based algorithm that first orders the vertices recursively by deleting minimum degree vertices, and then assigns colors in the reverse "smallest last" order [15]; and Distributed Lexicographic (DLEX), a distributed geometry-aware algorithm that assigns colors distributively respecting the order of the vertices' x coordinates (with ties broken according to the y coordinates) [11]. i_{ILP}, β_{ILP}, i_{SL}, β_{SL}, i_{DLEX} and β_{DLEX} represent the minimum and maximum size among all independent dominating sets obtained in the IDP partition solution by the ILP model, SL and DLEX coloring methodology respectively. In Table 2, we observe that coloring heuristics pack more vertices in any single set than ILP, i.e. β_{SL} and β_{DLEX} are closer to the upper bounds $\beta_0(G)$ or β_0^{tr} than β_{ILP} is. On the other hand, in any independent domatic partition produced by ILP, the size of the set of minimum cardinality i_{ILP} is much closer to the independent domination number $i(G)$ than the minimum values i_{SL} and i_{DLEX} obtained by the coloring algorithms.

Table 1: Domination chain values.

$\delta, n, \overline{r}$	i^{tr}	$\gamma(G)$	$i(G)$	$\beta_0(G)$	β_0^{tr}	$\beta_n(r)$
5,50,0.41	2.40	3.65	3.65	7.20	7.20	12.1
5,100,0.30	4.70	6.10	6.10	12.6	14.3	20.3
5,200,0.19	10.8	12.1	12.2	26.2	32.4	39.6
5,400,0.14	18.4	19.9	19.9	44.5	55.3	62.7
5,800,0.10	40.4	40.4	40.4	91.0	121	126
10,50,0.52	1.40	2.50	2.55	4.90	4.28	8.50
10,100,0.38	2.60	4.00	4.00	8.65	7.95	13.1
10,200,0.26	5.70	7.15	7.15	16.4	17.2	23.5
10,400,0.19	10.5	12.6	12.6	29.3	31.8	38.9
10,800,0.13	23.2	25.2	25.2	59.9	69.8	76.9
20,50,0.72	0.70	1.15	1.15	3.90	2.20	5.70
20,100,0.51	1.40	2.80	2.80	5.50	4.30	8.67
20,200,0.37	2.80	4.10	4.10	9.95	8.50	13.8
20,400,0.26	5.50	7.30	7.30	17.4	16.7	23.0
20,800,0.18	11.4	14.3	14.3	35.7	34.3	41.6

Table 2: Min/Max independent dominating sets sizes.

δ, n	i_{ILP}	β_{ILP}	i_{SL}	β_{SL}	i_{DLEX}	β_{DLEX}
5,50	3.90	5.10	4.55	6.10	4.40	6.45
5,100	7.20	9.05	9.50	10.9	8.85	11.8
5,200	14.5	17.7	19.7	22.0	20.5	23.4
5,400	23.1	26.5	34.7	37.2	35.7	40.4
5,800	47.1	52.4	74.8	77.3	77.3	82.5
10,50	2.60	3.90	2.80	4.35	2.80	4.65
10,100	4.20	6.10	5.45	7.40	5.40	8.05
10,200	7.90	10.2	11.6	13.7	10.8	14.8
10,400	13.6	16.6	21.4	24.5	21.2	26.5
10,800	28.7	33.1	45.9	49.6	47.0	52.6
20,50	1.15	3.00	1.15	2.95	1.15	3.25
20,100	2.85	3.95	3.10	4.85	3.15	5.35
20,200	4.15	6.30	5.85	8.45	5.85	9.30
20,400	7.65	10.2	11.7	14.9	11.5	16.2
20,800	15.7	20.4	23.1	27.5	22.4	29.3

Table 3: Non independent domatically full instances.

$(5, 100)$	$(5, 800)$	$(10, 50)$	$(20, 50)$	$(20, 100)$
95%(1)	90%(1)	95%(1)	40%(1,6)	85%(1)

4.2 Independent Domatic Partitions in RGGs

We report that all 290 experimented RGG instances were domatically full and 271 (over 93%) were independent domatically full (IDF). Namely, the cases $(5, 50)$, $(5, 200)$, $(5, 400)$, $(10, 100)$, $(10, 200)$, $(10, 400)$, $(10, 800)$, $(20, 200)$, $(20, 400)$ and $(20, 800)$ were all IDF. Table 3 shows the (δ, n) pairs where some instances are not IDF. For each (δ, n) pair, we report the percentage of random instances that are IDF, the second value(s) between parentheses denotes the number of sets (or min and max number of sets) that are missed compared to the upper bound $(\delta + 1)$. For example, in the $(\delta = 20, n = 50)$ case, 40% of the 20 instances were IDF, the lowest gap from $\delta + 1$ is one set, and the highest is 6 sets. The pattern we observe is that when δ is very close to n (a highly dense graph), the graph has a higher chance not to be independent domatically full.

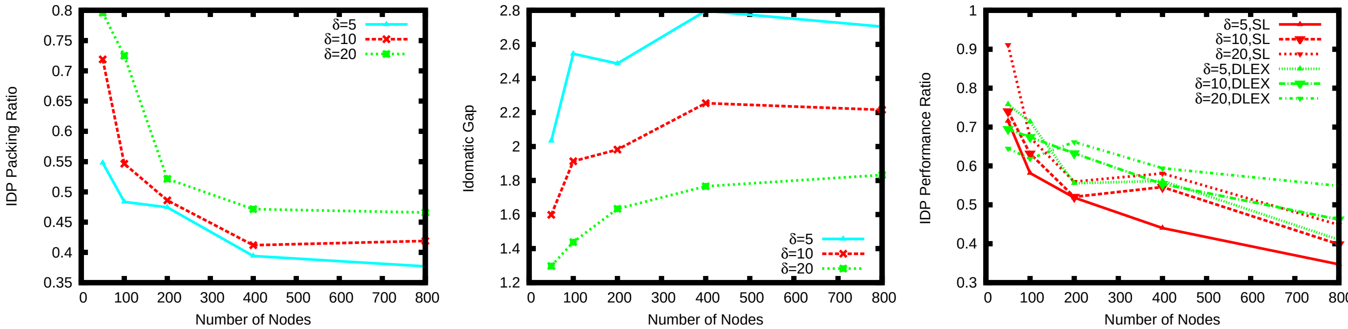
We define the IDP packing ratio as the portion of nodes of V in the $d_{ind}(G)$ independent dominating sets. Figure 1a shows the evolution of the ratio as n grows for various δ. For a fixed δ, the ratio decreases with increasing n, and it increases for fixed n as δ increases. We derive from [2] that if $\chi(G) \geq \omega(G) > \delta(G)+1$ then G is not idomatic. We use $\omega_{SL}(G)$ as a lower bound on the clique number obtained by Smallest Last coloring [12] and report that in all samples, $\omega_{SL}(G) > \delta(G) + 1$, therefore the graphs are not idomatic. We also confirm this observation by solving the ILP model of the Idomatic partition problem. We define the Idomatic gap as the ratio of the maximal clique value ω_{SL} over $\delta + 1$ and conjecture that the closer the ratio is to 1, the more likely the graph is to be idomatic. We observe that the Idomatic gap is correlated with the IDP packing ratio. Intuitively, the larger the Idomatic gap is, the lower is the IDP packing ratio. Figure 1b shows the evolution of the Idomatic gap as n grows for various δ. Figure 1c shows the performance ratio on $d_{ind}(G)$ obtained by SL and DLEX. We observe that the ratio decreases as n increases and it is generally higher for the same n when δ increases. Notice that these ratios are obtained as a by-product of the coloring algorithms whose purpose is unrelated to approximation of the IDP problem.

4.3 Independent Dominating Sets Properties

The main target application of this research is Wireless Sensor Networks' sleep scheduling. In that context, the generated independent dominating sets constitute candidate virtual backbones that can be rotated through to achieve efficient routing and data dissemination while minimizing individual sensors' energy expenditure and extending the network's lifetime. For a candidate backbone to be useful for routing, it has to be connected. Therefore, every generated independent dominating set has to be made connected [12, 11].
In any initial independent dominating set, a virtual backbone edge (or link) between two independent backbone nodes u and v is constructed if the following two rules are satisfied: there has to be at least one common neighbor w to u and v so that w may act as a relay node for message forwarding, and the edge has to satisfy the Gabriel graph rule to avoid edge cross-over and so that we obtain a planar connected virtual backbone link graph [12, 11]. Note that each edge (link) has a normalized link length αr with $1 < \alpha < 2$.

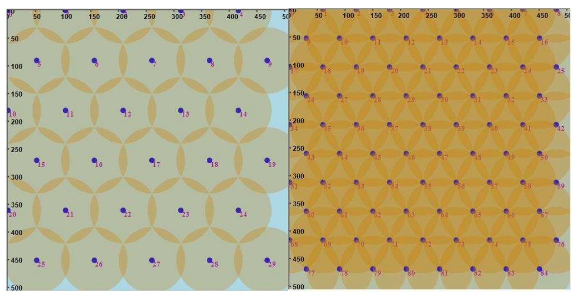
An interior edge (u, v) (denoted in light blue in Figures 3, 4, 5 and 6) is an edge that is incident to two triangles, i.e. node u can forward data to v directly through (u, v) and also via two alternative 2-hop paths (u, a, v) and (u, b, v) where a and b are two backbone nodes that are adjacent to both u and v in the backbone link graph. A boundary edge (u, v) (in dark blue) is an edge incident to one triangle, i.e. the edge is incident to two backbone nodes u and v that may reach each other directly through the edge (u, v) or via a third backbone node w that forms the triangle (u, w, v). A bridge edge (u, v) or simply bridge is an edge that is not incident to any triangle, i.e. node u may reach node v only via (u, v). This classification of links is related to the number of different alternative paths in an ideal case that any two adjacent backbone nodes may use to communicate, i.e. flexibility of communication between backbone neighbors. Intuitively, an interior edge may provide 3 possible disjoint paths (two 2-hop paths and one 1-hop direct path), a boundary edge provides 2 and a bridge only one direct 1-hop path.

To measure the quality of any single backbone generated by ILP or the coloring algorithms, it is relevant to compare the resulting backbone with the triangular lattice obtained by a regular closest packing. Figure 2 shows the regular triangular lattice packing extreme cases. Figure 2a has nodes placed at a normalized link distance $\sqrt{3} = 1.73$, which is the maximum separation allowing for independent domination of the plane by such a regular lattice. Figure 2b illustrates the closest packing lattice allowing for independent domination with normalized link length between nodes just over unity. We consider the following uniformity measures derived from the triangular lattice packing as introduced in [12]: link length uniformity expressed by the *Median Edge Length*, degree uniformity expressed by the *Average Degree*, triangle face uniformity expressed by the *Number of Triangles* and the existence of alternative disjoint 2-hop paths between every two backbone nodes expressed by the *Number of Interior Edges*.



(a) IDP Packing Ratio (b) Maximal Clique to $\delta+1$ ratio (c) IDP Perf. Ratio for various δ

Figure 1: Performance of the Independent Domatic and Idomatic Partitions for various δ.



(a) Sparsest Independent Dominating Lattice (b) Densest Independent Dominating Lattice

Figure 2: Extremal Triangular Lattice cases.

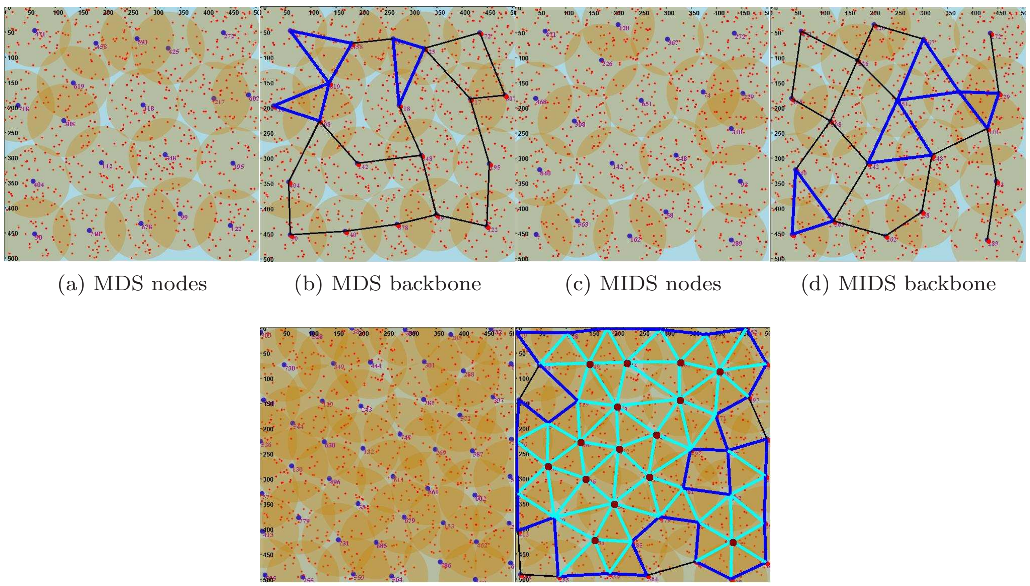
In this section, we examine in more detail one sample random geometric graph $G(n, r)$ where $n = 800$, $r = 0.146$ and δ =10. Based on r's value, $i^{tr} = \frac{1}{3}.[1/(r^2\frac{\sqrt{3}}{2})] = 18.05$ and $\beta_0^{tr} = 1/(r^2\frac{\sqrt{3}}{2}) = 54.17$. Table 4 shows the properties of the sets obtained by optimally solving the MDS, MIDS and MaxIS problems with ILP as well as the asymptotic extremal cases obtained by a triangular lattice packing. In this particular sample RGG, $\gamma(G) = 20$, $i(G) = 20$ and $\beta_0(G) = 49$. As observed before in Table 1, i^{tr} is a lower bound on $\gamma(G)$ and $i(G)$ and β_0^{tr} is an upper bound on β_0. Notice that in the triangular lattice (both sparse and dense packing) as the number of vertices $|V|$ tends to infinity, the number of edges $|E|$ tends to $3|V|$ since each vertex is incident to 6 edges and each edge is incident to 2 vertices. The average degree tends to 6, and the number of triangles tends to $2|V|$ since a vertex is incident to 6 triangles and a triangle is incident to 3 vertices. The number of interior edges also tends to $3|V|$ because when $|V| \to \infty$, the percentage of edges that touch the boundary goes down. In other words, the boundary effect tends to disappear and all edges tend to become interior edges.

Figure 3 shows that even though an MDS or MIDS-based backbone solution provides full node domination with the minimum number of nodes, it provides poor quality (high median edge length, a low average degree, very low number of triangles, no interior edges and a lot of bridges) in terms of redundant paths for efficient routing. Conversely, an MaxIS solution assures full node domination with the maximum number of nodes which may seem wasteful at first, but it offers high quality in terms of geometric regularity (number of nodes close to the asymptotic upper bound, high number of edges, low median edge length, average node degree close to a planar triangular lattice-based value of 6, high number of triangles, and a high number of interior edges). Geometric regularity is a desired property that translates in practice into a higher redundancy in terms of edge disjoint paths and a lower routing stretch factor.

Tables 5, 6 and 7 show the contrasting results obtained by the Integer Linear Program (ILP) on one hand and Smallest Last (SL) and Distributed Lexicographic (DLEX) colorings on the other hand. The earlier observations carry over as follows: ILP provides an optimal IDP solution of (δ+1) independent dominating sets with 267 nodes packed in the partition. The sets however are very sparse and close in size to the size of an MIDS.

Table 4: Quality of Minimum Dominating Set (MDS), Minimum Independent Dominating Set (MIDS), Maximum Independent Set (MaxIS), Sparsest Independent Dominating Lattice (SIDL) and Densest Independent Dominating Lattice (DIDL).

Set	$\|V\|$	$\|E\|$	*Median Edge Length*	$\overline{d}$	*#Triangles*	*#Interior Edges*
MDS	20	26	1.55	2.6	3	0
MIDS	20	29	1.63	2.9	4	0
MaxIS	49	115	1.10	4.69	62	76
SIDL	18.05	$\rightarrow 3\|V\|$	1.73	$\rightarrow 6$	$\rightarrow 2\|V\|$	$\rightarrow 3\|V\|$
DIDL	54.17	$\rightarrow 3\|V\|$	1.00	$\rightarrow 6$	$\rightarrow 2\|V\|$	$\rightarrow 3\|V\|$

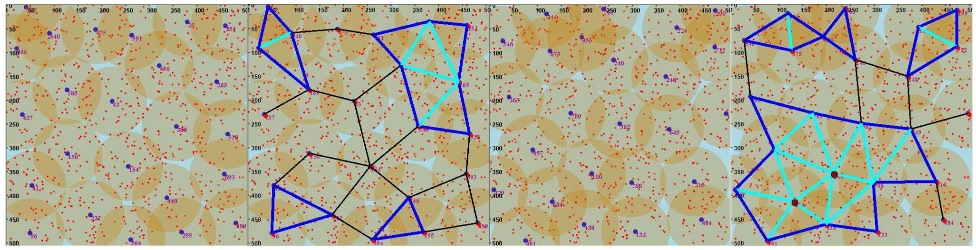


(a) MDS nodes (b) MDS backbone (c) MIDS nodes (d) MIDS backbone

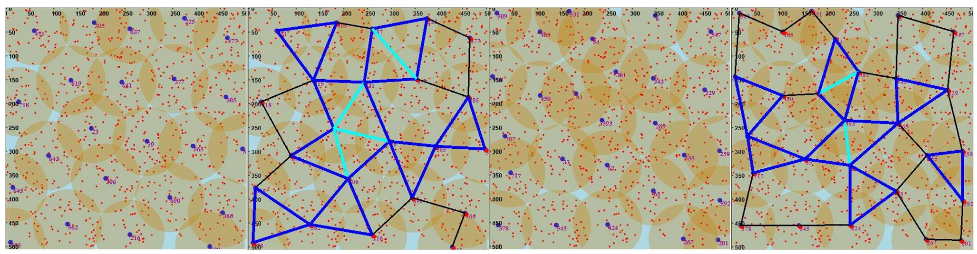
(e) MaxIS nodes (f) MaxIS backbone

Figure 3: Quality of Minimum Dominating Set (MDS), Minimum Independent Dominating Set (MIDS) and Maximum Independent Set (MaxIS), and their backbone link graphs.

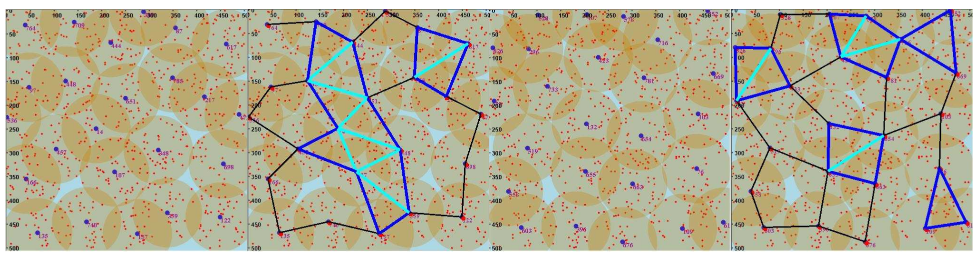
In Table 5, we report the quality measures of the sets: namely, a low number of edges $|E|$ in the order of $|V|$, a high median edge length close to $\sqrt{3}r$, a low average degree, a very low number of triangles, and a a very low number of interior edges. These results are confirmed graphically in Figure 4, where we observe the sparseness of the backbones and the low quality connectivity. On the other hand, SL coloring provides a sub-optimal IDP solution of only 6 independent dominating sets (compared to the optimal 11 sets) with 234 nodes packed in the 6 sets. However, the remaining 5 sets (among the first $(\delta + 1)$) are nearly dominating, and they are missing full domination by only a few nodes. In fact, the domination percentage of all first $(\delta+1)$ sets is 99.829% and the nearly dominating $(\delta + 1)$ partition packs 425 nodes. Furthermore, the sets obtained by SL are densely packed and close in size to the size of an MaxIS solution. They also offer high quality measures as reported in Table 6: high number of edges, low median edge length, an average degree close to the upper bound of 6, a high number of triangles and a high number of interior edges. This is further confirmed graphically in Figure 5 where we observe a better geometric regularity of the backbone sets than in the ILP solution. Similar good quality results are observed with Distributed Lexicographic coloring. We conclude from this study that despite the fact ILP delivers an optimal $(\delta+1)$ solution, it lacks the convenient feature of redundancy and geometric regularity obtained by the coloring algorithms. In addition, domination alone in this particular context may not be sufficient for practical applications of Wireless Sensor Networks.



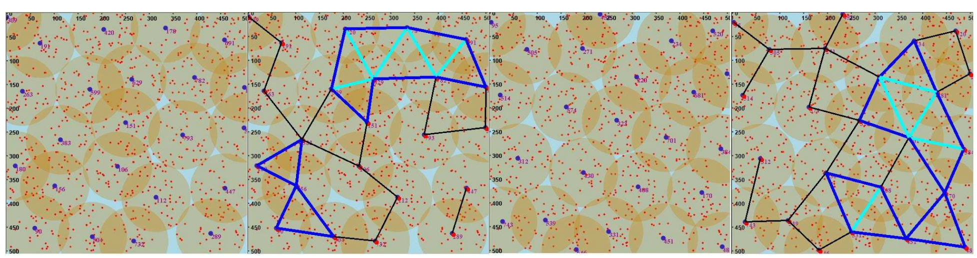
(a) Set 1 nodes (b) Set 1 backbone (c) Set 2 nodes (d) Set 2 backbone



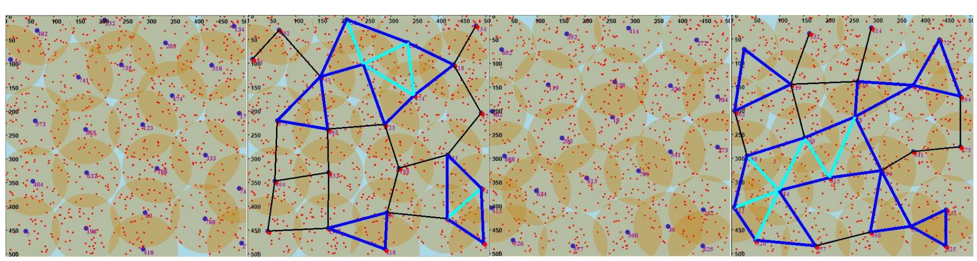
(e) Set 3 nodes (f) Set 3 backbone (g) Set 4 nodes (h) Set 4 backbone



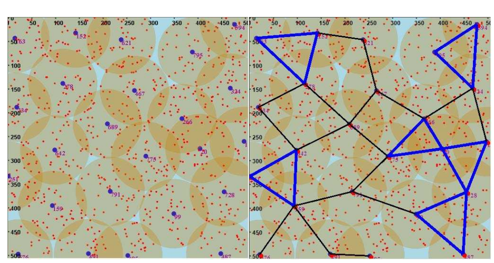
(i) Set 5 nodes (j) Set 5 backbone (k) Set 6 nodes (l) Set 6 backbone



(m) Set 7 nodes (n) Set 7 backbone (o) Set 8 nodes (p) Set 8 backbone

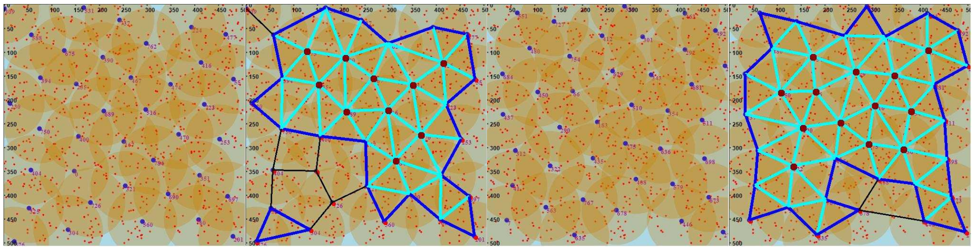


(q) Set 9 nodes (r) Set 9 backbone (s) Set 10 nodes (t) Set 10 backbone

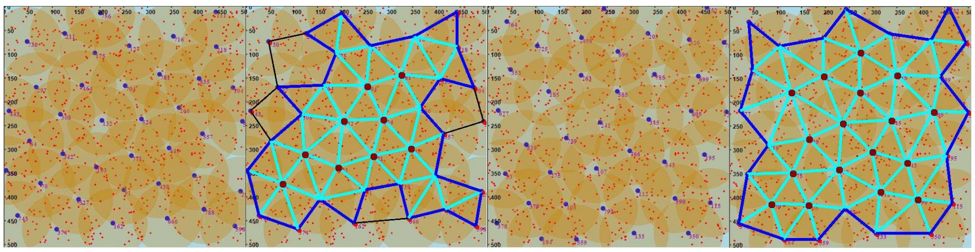


(u) Set 11 nodes (v) Set 11 backbone

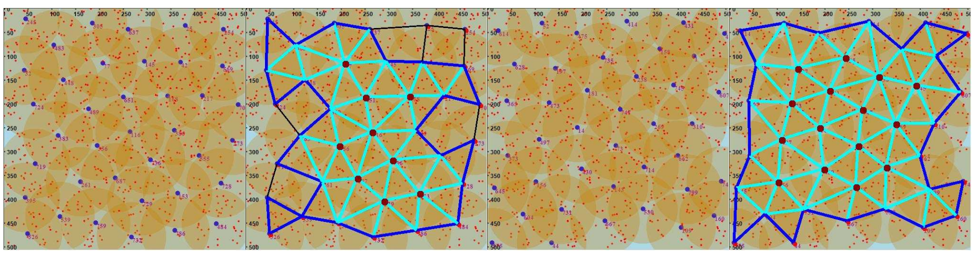
Figure 4: Quality of $(\delta+1)$ Independent Dominating Sets obtained by the Integer Linear Program, and their backbone link graphs.



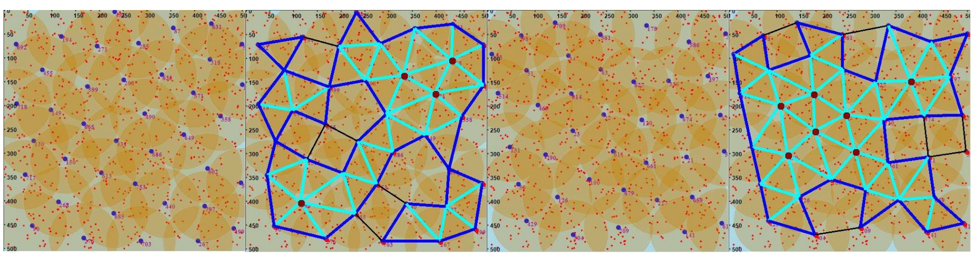
(a) Set 1 nodes (b) Set 1 backbone (c) Set 2 nodes (d) Set 2 backbone



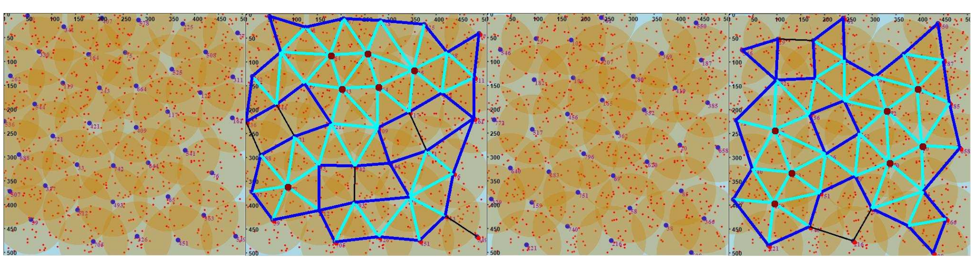
(e) Set 3 nodes (f) Set 3 backbone (g) Set 4 nodes (h) Set 4 backbone



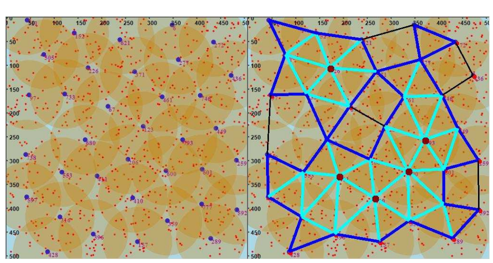
(i) Set 5 nodes (j) Set 5 backbone (k) Set 6 nodes (l) Set 6 backbone



(m) Set 7 nodes (n) Set 7 backbone (o) Set 8 nodes (p) Set 8 backbone

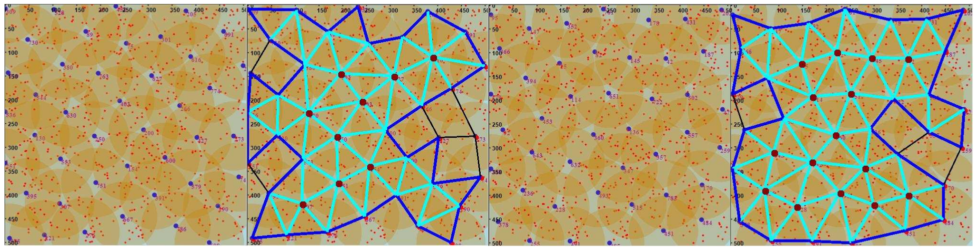


(q) Set 9 nodes (r) Set 9 backbone (s) Set 10 nodes (t) Set 10 backbone

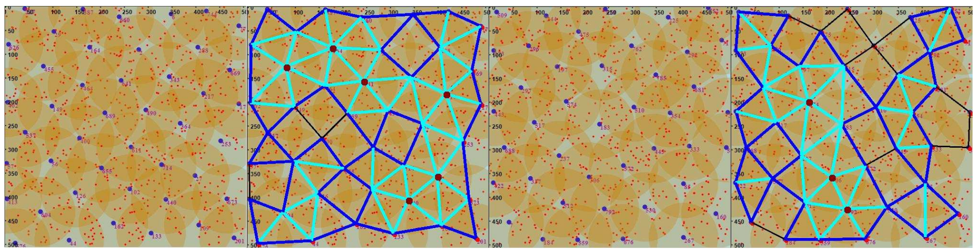


(u) Set 11 nodes (v) Set 11 backbone

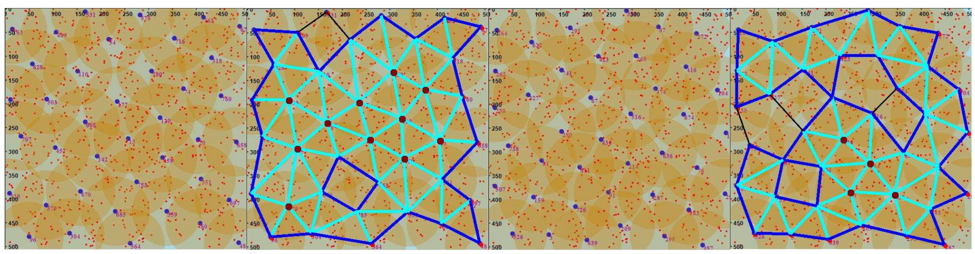
Figure 5: Quality of $(\delta + 1)$ Independent Dominating Sets obtained by Smallest Last coloring, and their backbone link graphs.



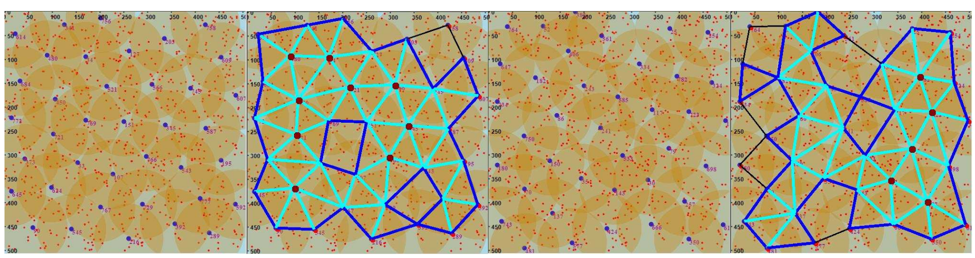
(a) Set 1 nodes (b) Set 1 backbone (c) Set 2 nodes (d) Set 2 backbone



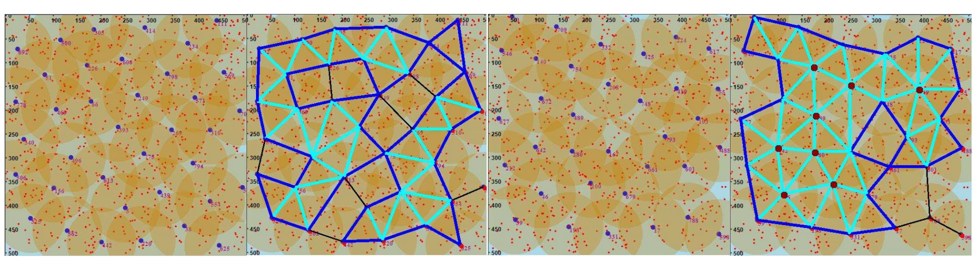
(e) Set 3 nodes (f) Set 3 backbone (g) Set 4 nodes (h) Set 4 backbone



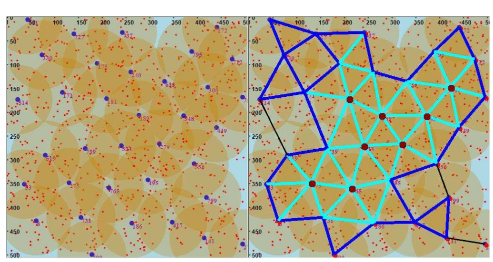
(i) Set 5 nodes (j) Set 5 backbone (k) Set 6 nodes (l) Set 6 backbone



(m) Set 7 nodes (n) Set 7 backbone (o) Set 8 nodes (p) Set 8 backbone



(q) Set 9 nodes (r) Set 9 backbone (s) Set 10 nodes (t) Set 10 backbone



(u) Set 11 nodes (v) Set 11 backbone

Figure 6: Quality of $(\delta + 1)$ Independent Dominating Sets obtained by Distributed Lexicographic coloring, and their backbone link graphs.

Table 5: Quality of $(\delta + 1)$ Independent Dominating sets produced by the Integer Linear Program.

Set	$\|V\|$	$\|E\|$	*Median Edge Length*	$\overline{d}$	*#Triangles*	*#Interior Edges*
1	24	38	1.52	3.16	9	5
2	25	44	1.50	3.52	18	16
3	23	39	1.60	3.39	11	4
4	27	43	1.51	3.18	9	2
5	24	37	1.50	3.08	10	8
6	25	39	1.55	3.12	9	4
7	23	32	1.47	2.78	8	5
8	24	36	1.60	3.00	8	5
9	24	38	1.52	3.16	9	5
10	24	39	1.57	3.25	12	6
11	24	35	1.57	2.91	6	0
Average	24.27	38.18	1.53	3.14	9.90	5.45

Table 6: Quality of $(\delta + 1)$ Independent Dominating sets produced by Smallest Last coloring.

Set	$\|V\|$	$\|E\|$	*Median Edge Length*	$\overline{d}$	*#Triangles*	*#Interior Edges*
1	38	80	1.12	4.21	40	48
2	40	92	1.11	4.60	51	63
3	39	85	1.13	4.35	43	51
4	39	92	1.17	4.71	54	70
5	38	84	1.14	4.42	42	49
6	40	92	1.14	4.60	53	67
7	39	83	1.13	4.25	39	39
8	38	84	1.12	4.42	42	48
9	41	91	1.12	4.43	45	49
10	37	81	1.12	4.37	42	48
11	36	78	1.16	4.33	37	39
Average	38.63	85.63	1.13	4.42	44.36	51.90

5 Conclusion and Future Work

We have shown experimentally that RGGs are domatically full in all instances and independent domatically full in 93% of the instances. Strongly chordal (SC) graphs are provably domatically full [3, 7]. Further research related to this work includes the problem of determining whether the experimented graphs are strongly chordal which would explain their domatic fullness. A more general question is are RGGs strongly chordal with high likelihood? Another direction we are pursuing is how do we enhance the coloring algorithms to improve their performance ratio in solving the IDP problem.

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Table 7: Quality of $(\delta+1)$ Independent Dominating sets produced by Distributed Lexicographic coloring.

Set	$\lvert V\rvert$	$\lvert E\rvert$	*Median Edge Length*	$\overline{d}$	*#Triangles*	*#Interior Edges*
1	43	96	1.16	4.46	49	57
2	43	101	1.16	4.69	56	70
3	44	100	1.14	4.54	50	53
4	42	87	1.15	4.14	37	35
5	39	89	1.20	4.56	49	60
6	40	88	1.15	4.40	44	47
7	38	87	1.17	4.57	47	56
8	39	84	1.15	4.30	40	43
9	37	78	1.14	4.21	34	31
10	34	74	1.22	4.35	39	46
11	35	75	1.16	4.28	38	42
Average	39.45	87.18	1.16	4.40	43.90	49.09

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