# Computational Financial Derivatives - A Primer

**Ruppa K. Thulasiram***

Computational Financial Derivatives Laboratory
**Technical Memo 00**

Department of Computer Science
University of Manitoba
Winnipeg, MB R3T 2N2, Canada

Sep.1, 2000

## DRAFT - PLEASE DO NOT DISTRIBUTE

*tulsi@cs.umanitoba.ca

**Abstract**

This memo is written to introduce the subject of computational finance generally and the topic of option pricing particularly. We introduce several terms related to option pricing and define them. A basic growth rate formula is derived. Using the concept of risk-free portfolio that yields at least what a growth rate analysis yields, which is a central idea, we derive Black-Scholes Model. We discuss the properties of the Black-Scholes equation and describe many possible methods of solving this equation. We also present a brief overview of the methods used in the finance literature with a survey of recent literature.

The purpose of this memo is to capture an overview of the fundamentals of financial derivatives and state-of-the-art option pricing methodologies, quickly, with a computational flavor. Appropirate solution methodologies for multithreaded computation is planned to be studied in the near future.

# Contents

**1 Introduction** **1**

**2 Definitions** **3**

**3 Growth Rate Formula** **5**

**4 Option Pricing Models: General Features** **6**

**5 Black-Scholes Model** **7**
5.1 General Assumptions of the Model . . . . . 7
5.2 Derivation of Black-Scholes Model . . . . . 8
5.2.1 Ito's Lemma . . . . . 8
5.2.2 Construct a Portfolio . . . . . 9
5.2.3 Basic Return . . . . . 9

**6 Solution Techniques** **10**
6.1 Finite Difference Methods . . . . . 11
6.2 Finite Element Methods . . . . . 11
6.3 Monte Carlo Technique . . . . . 11
6.4 **Lattice Methods** . . . . . 12

**7 note-conclude** **12**

**8 Further Reading** **12**

# 1 Introduction

Sequential computers performing one operation at a time are quickly reaching a physical limit beyond which their data process speed cannot be increased through the use of faster components. This limit is imposed by the speed of light in vacuum. However, in great many computational problems, the time taken to obtain a solution using a sequential computer is unacceptably slow. These problems arise in such diverse fields as weather forecasting, biomedical analysis and management of huge knowledge bases. One way out of this impasse is provided by parallel computation.

For decades, computer architects have incorporated parallelism into various levels of hardware in order to increase the performance of computer systems. To achieve the extremely high speeds demanded by contemporary science, architectures must now incorporate parallelism at the highest levels of the system. Today, the fastest computers in the world use high-level parallelism. These computers are leading to new scientific discoveries.

The current state-of-the-art "**grand challenges**" lists problems only from science and engineering [17, 1]; The problems facing **finance industry** has not been addressed in this category. The nature of the problems in finance industry also demand high-speed computing and efficient algorithms in solving problems such as option pricing, risk analysis, portfolio management etc.; they also require real-time solutions for some of finance applications such as data mining, estimation etc. We intend to address this issue through the current work.

Economic and financial models used for evaluation and forecasting purposes are generally large dynamic, linear or nonlinear, models that have to be solved for a certain time span. Model sizes upto 1000 equations are no exception. There are even examples of larger models, with more than 100,000 equations. Solving these models is, due to their size, computationally intensive. For this type of problems a supercomputer make it possible to generate model solutions in a resonable processing time. When the model is not too large it is even possible to produce results in a real-time fashion. Examples of this type of applications can be found in [2, 9, 13].

A very recent application is the modelling in connection to financial markets, like option market, stock market and the bond market. One of the main characteristics of financial markets is that changes in these market can take place very rapidly. Black Monday (October 19, 1987) has shown that in a very short time, asset prices may change significantly. In order to respond adequately on changes in the market it is necessary for large market participants, like brokers and banks, to evaluate information as fast as possible. This is necessary to make, for instance, the appropirate portfolio changes. Any delay in information access would mean a financial loss. According to the methodlogy developed by Black and Scholes, it is possible to approximate the implication of market changes on portfolio positions. Their method requires, however, solution of a set of partial differenential equations by means of numerical integration. Depending on the number of assets taken into account, obtaining a solution can be highly computationally intensive. For this application supercomputers can make it possible to have online information processing, making instantaneous response possible. Very recent applications are introduced in [8, 7].

**Financial Issues:**

The constant flow of information in the business world has beaten a path for the inroads of high performance computing. A number of applications are well suited to utilizing supercomputing resources: For example,

- Financial Instrument Pricing. Although some types of financial instruments have well-formed procedures for determining their values, there are exotic and interesting instruments that can be priced only by using a probabilistic method that performs an analysis of multiple simulations.
- Data Mining Applications: The proliferation of computer systems over the last 20 years, in both mainframe and workstation models, has allowed huge databases to be collected. The amount of data available has surpassed the ability of mainstream computation systems to process that data. In some instances, queries on databases with millions of records may take days or even weeks to complete.

**Parallel Computation Models**

Unlike the case in sequential computation, a wide variety of models has been proposed and used to study parallel computation in theory and to build parallel computers in practice. These models differ according to whether the processors communicate among themselves through a shared memory or distributed memory, whether the interconnection is in the form of an array, a tree or a hypercube, whether the processors execute the same or different algorithms, whether the processors operate synchronously or asynchronously, and so on. This diversity sets parallel computation apart as a rich and exciting field for practitioners and researchers alike; Furthermore, the computers are divided into vector computers, parallel computers in terms of the functionality of their architectures.

Among various such parallel computers, **EARTH** (Efficient Architecture for Running THreads) has been evolved from the data flow model of computation. However, it differs from its dataflow predecessors in that instructions are not synchronized and scheduled individually, but are combined into larger units called **threads**. A thread is a segment of code which is executed sequentially (although control-transfer instructions, such as branches, are allowed), and synchronized externally. The EARTH model is described in Appendix 1, and more detailed technical descriptions can be found in [16].

We, at the CAPSL (Computer Architectures and Parallel Systems Laboratory) of University of Delaware, have initiated the process of applying our EARTH computational platform to non-traditional applications such as the Computational Finance and Data Mining.

Our preliminary study of the vast field of finance in the past has indicated that many models used in finance, both industry and academia, end up in formulation of highly mathematical problems (for example, fundamental Black-Scholes model for option pricing ends up in a stochastic differential equations for the determination of option pricing). Solving these equations exactly in closed form is impossible as the experience in other fields suggests. Therefore, one has to look for numerical simulations; some work has already been reported in the

computational finance literature using high performance systems, though countable, indicating that the researchers are recognizing that problems in financial field are highly numerical and demanding high performance computing in addition to requiring efficient algorithms. This work is the first and unique attempt, to the knowledge of the authors, in studying the finance problem in a *multithreaded computational environement*.

## 2 Definitions

There are several areas of research in finance, both in industry and academia. Though not exhaustive, important ones can be categorized under

- **Option Pricing**
- **Risk Analysis**
- **Portfolio Management**
- **Data Mining**
- **Estimation**
- **Interest Rate Modeling**
- **Volatility and Inverse Problems**

among the different financial markets such as:

- **Stock or Shares or Equities Markets**
- **Bond Markets**: Government and other bonds
- **Currency or Foreign Exchange Markets**
- **Commodity Markets**: Oil, Gold, Copper, Electricity
- **Futures and Options Markets**: Complex Derivative products

Various terms involved in above markets and research areas are described in this section with examples. To be more comprehensive, or as a case study, only the terms related to **Option Pricing** is focused in this work.

The terms Financial Derivatives, Derivative Securities, Derivative Products, Contingent Claims or just Derivatives all are interchangeably used in the literature, both industry and academia.

**Definition 2.1:European Call Option:** At a prescribed future time (**Expiration date**) the **holder** may purchase a prescribed asset, known as **underlying asset** for a prescribed amount known as the **exercise price or strike price** The other party is known as **writer**. Issues at hand in this case include:

- The cost for the **holder's** right

- Minimizing the risk associated with **writer's** obligation .

**Call Option - an Example:** Let's say, today is Oct.2, 1998. With CALL OPTION one enters a deal to buy a stock at \$50 on May 2, 1999. On May 2, 1999 say the stock price is \$70. On the exercise day (May 2) the holder can exercise his option of buying the stock at \$50; and by immediately selling in the open market at \$70 the holder can THEREFORE HAVE A GAIN OF \$20. On May 2, 1999 if the stock price is \$30 the holder is not obligated to buy this (why to buy something at a higher price when the same is available at a lower cost in the open market). The question then is why would anyone write an option? The answer is the writer wants to make a profit by taking a **view** on the market.

**Definition 2.2**: **Speculation** Selling shares that one does not own is called selling **short position**. The opposite is a **long position**.

Writers are taking a **short position -** they expect the value to fall.

**Definition 2.3:** If these people are not there, the market goes only one direction which is an impractical scenario. These people are called **bears** and they are potential customers for **Put Options**.

**Definition 2.4:** Other group want the price to rise. They are called **bulls** and they are potential writers of Put options and buyers of Call options.

**Definition 2.5:Hedging** One has to reduce the risk. Taking advantage of the good correlations between asset and options is called **hedging.** If a market maker can sell an option for more than it is worth and then hedge away all the risk for the rest of the option's life, he has locked in a guaranteed, risk-free profit. This idea is central to the theory and practice of option pricing. (For example, on Oct.1, 1998 there was huge mis-mangement in handling in hedge funds and the UBS bank of Switzerland lost more than \$3 billion and the President and the Chief Executive Officer were sacked).

**Types of Options:**

- Call and Put Options discussed above form a small section of derivative products.
- Options described earlier which may only be exercised at maturity are called **European.**
- **Definition 2.6:American Option** is an option that may be exercised at any time prior to expiry. American Options are analogous to **Free Boundary Problems**, a mathematically challenging problem since there is no set boundary to refer to or iterate on.
  - Issue here is determining the best time to exercise the option.
- **Definition 2.7:Exotic Options** have values which depend on the history of an asset price. Exotic Options could be analogous to **crack propagation problem**, where damage at one particular point on a material depends on the distance of the crack source and its propagation along the material surface and interior.

- **Definition 2.8:Barrier Options -** The options can either come into existence (**knock-in**) or become worthless(**knock-out**) if the underlying asset reaches some prescribed value before expiry.
- **Definition 2.9:Asian Options -** the price depends on some form of average.
- **Definition 2.10:Lookback Options -** the price depends on the asset price maximum or minimum

**Definition 2.11:Forward Contract** is same as call option but with no element of choice

- There is no exchange of money taking place until delivery of the contract; that is, no premium is involved in this contract.
- Also there is no cost to enter the forward contract.
- This is set up between two individual parties.

**Definition 2.12:Futures Contract** is technically slightly different from Forward Contract in that:

- The value of future contract is evaluated every day and change is paid everyday.
- The net profit or loss is paid gradually across over life time of the contract.
- Future Contracts are traded on an exchange which specifies the standard features of these contracts.

# 3 Growth Rate Formula

In this section we derive the basic **Growth Rate** formula that forms a fundamental to option pricing problems.

The issue here is: How much should one pay now (**Present Value or Discounting**) for a guaranteed return at a particular future time. The growth rate can be derived as an exponential function as shown below.

The formula for simple interest I is: P*r*t, where P is the principal money; r, the interest rate; and t the period of time invested, that is,

$$I = Prt$$

i.e.,

$$dP = Prdt$$

Or

$$dP/P = rdt \tag{1}$$

The solution for this is

$$P = ce^{rt} \tag{2}$$

If the guaranteed future value at time t=T is known to be E

$$\mathbf{P = Ee^{-r(T-t)}} \tag{3}$$

If the interest rates are known function of time r(t) then the result is

$$P = Ee^{-\int_t^T r(s)ds} \tag{4}$$

Equations (3) or (4) will be used later in this note while deriving the celebrated equation based on Black-Scholes model.

# 4 Option Pricing Models: General Features

Most of the models for option price movement is based on historical or market data. Fundamental efficient market hypotheses comprises of the facts that:

- past history is fully reflected in the present price, which does not hold any further information.
- markets respond immediately to any new information about an asset.

Therefore, modeling involves arrival of new information which affects the price. The unanticipated asset price change due to these sudden information follow a *Markov Process*.

**Definition 4.1:Return** is the change in the asset price divided by the original value. That is, we are interested in a relative change (dS/S) rather to an absolute change. This change is divided into two parts:

1. **Definition 4.2:** Predictable, deterministic and anticipated return akin to the return on the money invested: $\mu$**dt**; $\mu$ is the average rate of growth of the asset price, **drift.**
2. **Definition 4.3:** Random change in response to external effects; represented as a random sample drawn from a normal distribution with mean zero: $\sigma$**dX**; $\sigma$ is **volatility**, a measure of the standard deviation of the returns.

Therefore, mathematical representation of simple generation of asset prices is:

$$dS/S = \mu dt + \sigma dX \tag{5}$$

If there is no randomness involved: dS/S = $\mu$dt which gives upon integration

$$S = S_0 e^{\mu(t-t_0)}$$

The term dX which is certainly a feature of asset prices and is said to follow a **Weiner process** with the following properties:

- dX is a random variable from a normal distribution.
- mean of dX = 0.
- variance of dX = dt.

# 5 Black-Scholes Model

This is a differential equation for the price of the simplest options called European vanilla options There are never any opportunities for risk-free profit to make an **instantaneous risk-free** profit. At least, **arbitrage** fees will consume part of the profit. **Arbitrage** is one of the fundamental concepts underlying the theory of financial derivative pricing and hedging. In other words, those opportunities cannot exist for a significant length of time before prices move to eliminate them. The financial application of this principle leads to some elegant modeling. *However, all financial* **theories** *assumes the existences of* **risk-free** *investments that give a guaranteed return with no chance of default, including the Black-Scholes Model.* For example, A good approximation to such an investment is a government bond or a deposit in a sound bank. *The greatest risk-free return that one can make on a portfolio of assets is the same as the return if the equivalent amount of cash were placed in a bank.* If that is not so, or in other words, if the yield from investment (other than bank) is higher than that of bank and is risk-free then everyone would put their money in portfolios. To avoid that happening, bank would increase the interest rates to attract more money/customers. The elasticity involving frictional factors such as transaction costs, differences in borrowing and lending rates, problems with liquidity, tax laws etc., always exists and within this **viscous** domain and the push and pull between bank investment and other investments **arbitragers** inhabit who seeks out and exploit irregularities or mispricings just mentioned above.

## 5.1 General Assumptions of the Model

The general assumptions of Black-Scholes Model are:

- The asset price follows the lognormal random walk
- The risk free interest rate **r** and the asset volatility $\sigma$ are known functions of time over the life of the option
- There are no transaction costs associated with hedging a portfolio
- No payment during the life of the option
- There are no arbitrage possibilities
- Trading of the underlying asset can take place continuously
- Short selling is permitted and the assets are divisible.

## 5.2 Derivation of Black-Scholes Model

The steps in derivation of the final equation involves:

- Start from a simple growth rate analysis with DRIFT and VOLATILITY
- Apply Ito's Lemma on the portfolio based on Option Value and Asset
- Arrive at the Black-Scholes Equation

### 5.2.1 Ito's Lemma

We first describe this lemma which will be used in this derivation. Ito's Lemma is for functions of Random variables like Taylor's expansion is for functions of deterministic variables.

Taylor expansion for a function f(S) is given by:

$$df = \frac{df}{dS}dS + \frac{1}{2}\frac{d^2f}{dS^2}dS^2 + ..... \tag{6}$$

$$dS^2 = (\sigma S dX + \mu S dt)^2 \tag{7}$$

$$= \sigma^2 S^2 dX^2 + 2\sigma\mu S^2 dt dX + \mu^2 S^2 dt^2 \tag{8}$$

The first term is the dominant term in this equation.

$$dX = O(sqrt(dt)$$

$$dS^2 = \sigma^2 S^2 dX^2$$

Since

$$dX^2 \longrightarrow dt, dS^2 \longrightarrow \sigma^2 S^2 dt$$

Therefore,

$$df = \frac{df}{dS}(\sigma S dX + \mu S dt) + \frac{1}{2}\sigma^2 S^2 \frac{d^2f}{dS^2}dt \tag{9}$$

Or,

$$df = \sigma S \frac{df}{dS}dX + \left(\mu S \frac{df}{dS} + \frac{1}{2}\sigma^2 S^2 \frac{d^2f}{dS^2}\right)dt \tag{10}$$

For functions of two variables f(S,t) the above equation can be shown to be

$$df = \sigma S \frac{\partial f}{\partial S}dX + \left(\mu S \frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + \frac{\partial f}{\partial t}\right)dt \tag{11}$$

Let V(S,t) be an Option. This could be a Call, Put or even a whole portfolio. Applying Ito's Lemma on this option gives:

$$dV = \sigma S \frac{\partial V}{\partial S} dX + \left( \mu S \frac{\partial V}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t} \right) dt \tag{12}$$

### 5.2.2 Construct a Portfolio

Now construct a portfolio involving assets and options as:

$$\pi = V - \Delta S \tag{13}$$

Jump in this portfolio at one-time step is

$$d\pi = dV - \Delta dS \tag{14}$$

Or

$$d\pi = \sigma S \left( \frac{\partial V}{\partial S} - \Delta \right) dX + \left( \mu S \frac{\partial V}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t} - \mu \Delta S \right) dt \tag{15}$$

Now eliminate the random component by choosing $\Delta = \frac{\partial V}{\partial S}$ ($\Delta$ is the value of $\frac{\partial V}{\partial S}$ at the start of time-step dt.

$$d\pi = \left( \frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \right) dt \tag{16}$$

### 5.2.3 Basic Return

A basic Return on an amount $\pi$ in risk-less asset is:

$$r \pi dt$$

Therefore, the change in the value of the portfolio with low cost arbitrage has to be at least equal to the basic return. That is,

$$r \pi dt = \left( \frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \right) dt \tag{17}$$

Substituting earlier equations (13), (16) into the above equation 17:

$$\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0 \tag{18}$$

This is the basic Black-Scholes Partial Differential Equation.

# 6 Solution Techniques

**The Equation:** As there was no reference to a CALL or PUT or PORTFOLIO, the dependent variable V in this equation could refer to any of them and hence they all satisfy the B-S Equation. The equation is of **PARABOLIC** type for S $< 0$ and **HYPERBOLIC** type for S=0. The S=0 line forms important **barrier** in financial terms in which information cannot cross – (It would be an interesting and challenging scenario if we have a system which involves dynamically evolving conditions that could only be described by a coupled parabolic and hyperbolic system of equations). The B-S is a 2nd order PDE in S and first order t. Therefore, for a unique solution of this equation, we need 2 boundary conditions for S and one boundary condition on t. The fact that the coefficients of the highest derivatives in S and time 't' are positive, implies that the B-S is backward parabolic equation. This means a future time (expiration time ) has to be specified for the computation of the option value. This also implies that the solution domain is for $t < T$.

In this section we briefly described the different solution techniques available for solving the Partial Differential Equations (PDE). We also introduce different other techniques used in solving option pricing problems, in the finance literature. Numerical solutions become necessary when closed from solution can not be reached by analytical means. For non-linear PDEs, exact solutions can be obtained only for a small class of reduced problems using lots of assumptions on the problem, which simply modify or lessens the quality of the original problem studied. Transformation techniques have helped to a good extent in reducing the severity of the original problem and obtaining either numerical or analytical on the transformed equations. A similarity transformation technique can identify and group many different parameters of the problem into a single one thereby combining the effects of many such parameters into one. Therefore, single value of this parameter would effectively combine a vastly different combinations of different parameters of the problem. In addition, varying this single parameter would exhaustively cover the whole domain of possible values of the constituting parameters of the problem. If a similarity transformation can not be found for a problem, a numerical solution is the choice. Numerical solution of PDEs can be categorized into

- Finite Difference Methods (for example, [4])
- Finite Element Methods (for example, [19])
- Finite Volume Methods

These methods have not found much familiarity in the financial literature expect for a very few recent works mentioned in parenthesis. The following methods are widely used, however.

- Lattice Methods (Binomial, Trinomial Tress) (for example, [3], [14])
  - Continuous random walk is replaced with discrete random walk in eq.(1)
  - this is analogous to *Method of Characteristics* in Gas Dynamics.
  - This is suited better for American Options since this option can be exercised at any time before expiry: Free Boundary Problem.

- Monte Carlo: Few use this method (for example, [15], [11], [12])

### 6.1 Finite Difference Methods

These methods are widely used in many engineering disciplines and a well established subject. We just list some of the features of these methods.

- This methods form a potential candidate for financial applications.
- Formulation is reasonably straight forward.
- Discretization of the problem domain is almost uniform.
- Parallelization of a Option Pricing (Finite Difference) Code is shown to be faster by two orders of magnitude in CM2, DECMPP12000 machines compared to their serial version.
- Some work is also carried out on Cray T3D by Clark ([4])
- Other works include [6] and [5].

### 6.2 Finite Element Methods

These methods are widely used in mechanical engineering mostly to study materials.

- These methods are proven to be robust.
- This allows for the use of irregular triangular meshes that improve computational efficiency.
- Arbitrary constrains on the solution can be imposed.
- This method will accommodate adaptiveness in the simulation to account for the real time changes.
- Zvan(1998, 1998a) Pentium PC 233MHz
- Multithreading is proven to be effective using FEM.

### 6.3 Monte Carlo Technique

- This technique has been used for derivatives involving highly random parameters.
- With more sampling size accurate results are expected
- However, as the sampling size increases computational resource requirement increases
- Paskov 1997; Thomas Coleman 1998(Matlab on PC); Alexander Sokol 1998(Pentium 233 MHz under Windows NT);

### 6.4 Lattice Methods

- This includes Binomial, Trinomial approaches
- They are most widely used in finance
- The main difference between these methods and Finite Difference Methods is that these methods are inherently discrete. That means there are differential equation involved here to discretize before solving. In that sense, though they are viewed as distinct techniques they are in fact simple explicit finite difference schemes (Wang 1998: Pentium 233MHz)
- These methods are intuitive and simple for basic option pricing as we explain the one-step and two-methods in the next section.
- For complex option pricing problems such as path-dependent Asian Options, heuristic methods are often employed to reduce the computational complexities of lattice methods.

The possibilities of EARTH's potentiality to address the research issues in Finance in terms of new computational challenges is brought out through this work. At CAPSL, we are taking on these CFD problems more rigorously because we have something to offer: that is EARTH, a multithreaded high performance computing system. We strongly believe, EARTH can offer new opportunities, in addition to solving the finance problems such as the one studied in this work, to solve some such problem in real-time.

**Future Work**

The following is a list of problems being studied or planned to be carried out in the near-future.

- Increasing the complexity of the algorithm constraints such as
  - divident payment at every time step
  - variable up and down movement of the stock price at each time step
  - variable rate of interest and
  - varibale time step itself
- solving the Black-Scholes equation using Finite Difference Method on EARTH
- exploring Monte Carlo Simulation on EARTH

Also the Threaded-C code given in this node in appendix-2 is a very naive way of implementation. This implementation itself could be improved using many advanced features EARTH operations described in [16].

## 7 note-conclude

## 8 Further Reading

For further literature on the preliminaries of the subject area the readers are directed to [10, 18] among other books, journal articles and conference papers.

## References


[1] H. M. Amman. *Supercomputing in Economics: A New Field of Research?* High Performance Computing, (Ed: J.L. Delhaye, E. Gelenbe) Elsevier Science Publishers B.V. (North-Holland), 1989.

[2] A. Ando, P. Beaumont, and M. Ando. Efficiency of the cyber 205 for stochastic simulation of a simultaneous, nonlineardynamic econometric model. *International Journal of Supercomputers and their Applications*, 1987.

[3] P. Carr and G. Yang. Simulating american bond options in an hjm framework. *Proceedings of the first National Conference on Computational and Quantitative Finance*, Sep. 1998.

[4] I. J. Clark. Option pricing algorithms for the cray t3d supercomputer. *Proceedings of the first National Conference on Computational and Quantitative Finance*, Sep. 1998.

[5] M. Dempster and J. Hutton. *Numerical Evaluation of Cross Currency Swaps and Swaptions.* In Mathematics of Derivative Securities (Ed: Dempster, M.A.H. and Pliska, S.R.), Cambridge University Press, Cambridge, 1997.

[6] N. Ekvall. Two finite difference schemes for evaluation of contingent claims with three underltying state variables. Technical Report 6520, Ekonomiska Forskningsinstitutet, Stockholm, Norway, 1993.

[7] G. Foster. Bond pricing in apl: A study in numerical solution of the brennan and schwartz bond pricing model using a vector processos. *Computer Science in Economics and Management*, 1989.

[8] G. Foster, W. Foote, and J. Kreamer. Force: Financial options - rapid calculations and execution. Technical memo, Syracuse University, Syracuse, NY, 1988.

[9] P. Henoff and A. Norman. Solving nonlinear econometric models using vector processors. *Proceedings of the IFAC owrkshop on modeling Decision and Games with Applications to Social Phenomena*, 1986.

[10] J. C. Hull. *Options, Futures and Other Derivatives.* (3rd Edition) Prentice Hall, Upper Saddle River, NJ, 1997.

[11] G. J. Jiang and J. L. Knight. Finite sample comparison of alternative estimators of ito diffusion process: a monte carlo study. *Proceedings of the first National Conference on Computational and Quantitative Finance*, Sep. 1998.

[12] S. Paskov. *New Methodologies for Valuing Derivatives.* In Mathematics of Derivative Securities (Ed: Dempster, M.A.H. and Pliska, S.R.), Cambridge University Press, Cambridge, 1997.

[13] C. Peterson. *Computer Simulation of Large Scale Models, Project Link.* In Scientific Computing on IBM vector Multiprocessors (Ed: R. Benzi and P. Sguazzero), Volume II, ECSEC, 1987.

[14] S. Peterson, R. Stapleton, and M. G. Subrahmanyam. An arbitrage free two factor model of the term structure of interest rates: A multivariate binomial approach. *Proceedings of the first National Conference on Computational and Quantitative Finance*, Sep. 1998.

[15] A. Sokol. Adaptive monte-carlo and trees for multifactor models. *Proceedings of the first National Conference on Computational and Quantitative Finance*, Sep. 1998.

[16] K. B. Theobald, J. N. Amaral, G. Heber, O. Maquelin, X. Tang, and G. R. Gao. Overview of the Threaded-C language. CAPSL Tech. Memo 19, Dept. of Elec. and Computer Eng., U. of Delaware, Newark, Delaware, Mar. 1998. In ftp://ftp.capsl.udel.edu/pub/doc/memos.

[17] A. B. Tucker and E. in Chief. *Computer Science and Engineering Handbook*. CRC Press, Boca Raton, Florida, 1997.

[18] P. Wilmott, S. Howison, and J. Dewynne. *The Mathematics of Financial Derivatives*. Cambridge Univesrity Press, Cambridge, UK, 1995.

[19] R. Zvan, P. Forsyth, and K. Vetzal. A general finite element approach for pde option pricing models. *Proceedings of the first National Conference on Computational and Quantitative Finance*, Sep. 1998.