4. Mobile Facility Location

INPUT
Given a time interval, \( T = [t_0, t_f] \), and a set of client position functions, \( P = \{p_1, \ldots, p_n\} \), where, \( p_i = T \rightarrow \mathbb{R}^2 \),

OUTPUT
identify a position function for a facility, \( f = T \rightarrow \mathbb{R}^2 \).

5. Velocity

The velocity of \( p_i \) is bounded by \( v \) if
\[
\forall t_1, t_2, \quad \|p_i(t_1) - p_i(t_2)\| \leq v|t_1 - t_2|.
\]

Even when the velocity of clients is bounded, the Euclidean centre and Euclidean median move with unbounded velocity. Consequently, defining a mobile facility that moves with bounded velocity (relative to the clients’ velocities) requires approximation.

6. Approximation Factor

How well does a function approximate the Euclidean centre or the Euclidean median?

\[
\lambda(P) = \frac{\max_{p \in P} \|\lambda(P) - p\|}{\max_{p \in P} \|\lambda(P) - p\|} \quad \text{(Y is a centre function)}
\]

or
\[
\sum_{p \in P} \|Y(P) - p\| \leq \lambda \sum_{p \in P} \|M(P) - p\| \quad \text{(Y is a median function)}
\]

7. Goals

A mobile facility must balance two opposing goals:
1. maintain a low upper bound on maximum velocity, and
2. provide a good approximation factor.