New Hardness Results for Guarding Orthogonal Polygons with Sliding Cameras*

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Abstract

Let P be an orthogonal polygon. Consider a sliding camera that travels back and forth along an orthogonal line segment $s \in P$ as its *trajectory*. The camera can see a point $p \in P$ if there exists a point $q \in s$ such that pq is a line segment normal to s that is completely inside P. In the *minimum-cardinality sliding cameras problem*, the objective is to find a set S of sliding cameras of minimum cardinality to guard P (i.e., every point in P can be seen by some sliding camera) while in the *minimum-length sliding cameras problem* the goal is to find such a set S so as to minimize the total length of trajectories along which the cameras in S travel.

In this paper, we first settle the complexity of the minimum-length sliding cameras problem by showing that it is polynomial tractable even for orthogonal polygons with holes, answering a question asked by Katz and Morgenstern [8]. We next show that the minimum-cardinality sliding cameras problem is NP-hard when P is allowed to have holes, which partially answers another question asked by Katz and Morgenstern [8].

1 Introduction

The art gallery problem is well known in computational geometry, where the objective is to cover a geometric shape (e.g., a polygon) with visibility regions of a set of point guards while minimizing the number of guards. The problem's multiple variants have been examined extensively (e.g., see [12, 14]) and can be classified based on the type of guards (e.g., points or line segments), the type of visibility model and the geometric shape (e.g., simple polygons, orthogonal polygons [5], polyominoes [1]).

In this paper, we consider a variant of the orthogonal art gallery problem introduced by Katz and Morgenstern [8], in which *sliding cameras* are used to guard the gallery. Let P be an orthogonal polygon with n vertices. A sliding

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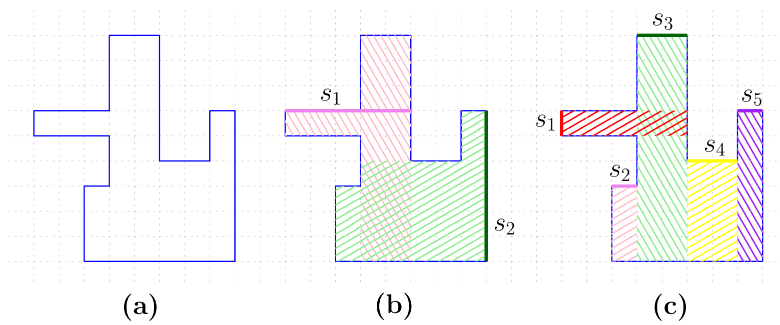


Figure 1: An illustration of the variants of the problem. Each grid cell has size 1×1. (a) A simple orthogonal polygon P. (b) An optimal solution for the minimum-cardinality sliding cameras problem on P. The trajectories of two sliding cameras s_1 and s_2 are shown in pink and green, respectively; each shaded region indicates the visibility region of the corresponding camera. This set of two cameras is an optimal solution to the minimum-cardinality sliding cameras problem on P. (c) A set of five sliding cameras whose total length of trajectories is 7, which is an optimal solution for the minimum-length sliding cameras problem on P.

camera travels back and forth along an orthogonal line segment s inside P. The camera (i.e., the guarding line segment s) can *see* a point $p \in P$ (equivalently, p is *orthogonally visible* to s) if and only if there exists a point q on s such that pq is normal to s and is completely contained in P. We study two variants of this problem: in the *minimum-cardinality sliding cameras (MCSC) problem*, we wish to minimize the number of sliding cameras so as to guard P entirely, while in the *minimum-length sliding cameras (MLSC) problem* the objective is to minimize the total length of trajectories along which the cameras travel; we assume that in both variants of the problem, polygon P and sliding cameras are constrained to be orthogonal. In both variations, every point in P must be visible to some camera at some point along its trajectory. See Figure 1.

Throughout the paper, we denote an orthogonal polygon with n vertices by P. Moreover, we denote the set of vertices and the set of edges of P by $V(P)$ and $E(P)$, respectively. We consider P to be a closed set; therefore, a camera's trajectory may include an edge of P. We also assume that a camera can see any point on its trajectory. We say that a set T of orthogonal line segments contained in P is a *cover of* P, if their corresponding cameras can collectively see any point in P; we sometimes say that the line segments in T *guard* P entirely.

Related Work. The art gallery problem was first introduced by Klee in 1973. Two years later, Chvatal [2] gave an upper bound proving that $\lfloor n/3 \rfloor$ point guards are always sufficient and sometimes necessary to guard a simple polygon with n vertices. The orthogonal art gallery problem was first studied by Kahn et al. [6] who proved that $\lfloor n/4 \rfloor$ guards are always sufficient and sometimes necessary to guard the interior of a simple orthogonal polygon. Lee and Lin [9] showed that the problem of guarding a simple polygon using the minimum number of guards is NP-hard. Moreover, the problem was also shown to be NP-hard for orthogonal polygons [13].

Limiting visibility allows some versions of the problem to be solved in polynomial time. Motwani et al. [11] studied the art gallery problem under s-visibility, where a guard point $p \in P$ can see all points in P that can be connected to p by an orthogonal staircase path contained in P. They use a perfect graph approach to solve the problem in polynomial time. Worman and Keil [16] defined r-visibility, in which a guard point $p \in P$ can see all points $q \in P$ such that the bounding rectangle of p and q (i.e., the axis-parallel rectangle with diagonal $\overline{pq}$) is contained in P. Given that P has n vertices, they use a similar approach to Motwani et al. [11] to solve this problem in $\widetilde{O}(n^{17})$ time, where $\widetilde{O}()$ hides poly-logarithmic factors. Moreover, Lingas et al. [10] presented a linear-time 3-approximation algorithm for this problem.

Recently, Katz and Morgenstern [8] introduce sliding cameras as another model of visibility to guard a *simple* orthogonal polygon P; they only study the MCSC problem. They first consider a restricted version of the problem, where cameras are constrained to travel only vertically inside the polygon. Using a similar approach to Motwani et al. [11] they construct a graph G corresponding to P and then show that (i) solving this problem on P is equivalent to solving the minimum clique cover problem on G, and that (ii) G is chordal. Since the minimum clique cover problem is polynomial solvable on chordal graphs, they solve the problem in polynomial time. They also generalize the problem such that both vertical and horizontal cameras are allowed (i.e., the MCSC problem); they present a 2-approximation algorithm for this problem under the assumption that the given input is an x-monotone orthogonal polygon. They leave open the complexity of the problem and mention studying the minimum-length sliding cameras problem as future work.

A *histogram* H is a simple polygon that has an edge, called the *base*, whose length is equal to the sum of the lengths of the edges of H that are parallel to the base. Moreover, a *double-sided histogram* is the union of two histograms that share the same base edge and that are located on opposite sides of the base. It is easy to observe that the MCSC problem is equivalent to the problem of covering P with minimum number of double-sided histograms. Fekete and Mitchell [3] proved that partitioning an orthogonal polygon (possibly with holes) into a minimum number of histograms is NP-hard. However, their proof does not directly imply that the MCSC problem is also NP-hard for orthogonal polygons with holes.

Our Results. In this paper, we first answer a question asked by Katz and Morgenstern [8] by proving that the MLSC problem is solvable in polynomial time even for orthogonal polygons with holes (see Section 2). We next show that the MCSC problem is NP-hard for orthogonal polygons with holes (see Section 3), which partially answers another question asked by Katz and Morgenstern [8].

We conclude the paper by Section 4.

2 The MLSC Problem: An Exact Algorithm

In this section, we give an algorithm that solves the MLSC problem exactly in polynomial time even when P has holes. Let T be a cover of P. In this section, we say that T is an *optimal cover for P* if the total length of trajectories along which the cameras in T travel is minimum over that of all covers of P. Our algorithm relies on reducing the MLSC problem to the *minimum-weight vertex cover problem* in graphs. We remind the reader of the definition of the minimum-weight vertex cover problem:

Definition 1. *Given a graph $G = (V, E)$ with positive edge weights, the* minimum-weight vertex cover *problem is to find a subset $V' \subseteq V$ that is a vertex cover of G (i.e., every edge in E has at least one endpoint in V') such that the sum of the weights of vertices in V' is minimized.*

The minimum-weight vertex cover problem is NP-hard in general [7]. However, it is solvable in polynomial time when the input graph is *bipartite* because the constraint matrix of the Integer Program corresponding to the minimum-weight vertex cover problem is totally unimodular [15]. Given P, we first construct a vertex-weighted graph G_P and then we show that (i) the MLSC problem on P is equivalent to the minimum-weight vertex cover problem on G_P, and that (ii) graph G_P is bipartite.

Similar to Katz and Morgenstern [8], we define a partition of an orthogonal polygon P into rectangles as follows. Extend the two edges of P incident to every reflex vertex in $V(P)$ inward until they hit the boundary of P. Let $S(P)$ be the set of the extended edges and the edges of P whose endpoints are both non-reflex vertices of P. We refer to elements of $S(P)$ simply as *edges*. The edges in $S(P)$ partition P into a set of rectangles; let $R(P)$ denote the set of resulting rectangles. We observe that in order to guard P entirely, it suffices to guard all rectangles in $R(P)$. The following observations are straightforward:

Observation 1. *Let T be a cover of P and let s be an orthogonal line segment in T. Then, for any partition of s into line segments $s_1, s_2, \ldots, s_k$ the set $T' = (T \setminus s) \cup \{s_1, \ldots, s_k\}$ is also a cover of P and the respective sums of the lengths of segments in T and T' are equal.*

Observation 2. *Let T be a cover of P. Moreover, let T' be the set of line segments obtained from T by translating every vertical line segment in T horizontally to the nearest boundary of P to its right and every horizontal line segment in T vertically to the nearest boundary of P below it. Then, T' is also a cover of P and the respective sums of the lengths of line segments in T and T' are equal. We call T' a* regular cover *of P.*

We now prove the following result.

Lemma 1. *Let $R \in R(P)$ be a rectangle and let T be a cover of P. Then, there exists a set $T' \subseteq T$ such that all line segments in T' have the same orientation (i.e., they are all vertical or they are all horizontal) and they collectively guard R entirely.*

Proof. Suppose, by a contradiction, that there does not exists such a set T'. Let R_v (resp., R_h) be the subregion of R that is guarded by the all union of the vertical (resp., horizontal) line segments in T and let $R_v^c = R \setminus R_v$ (resp., $R_h^c = R \setminus R_h$). Since R cannot be guarded exclusively by vertical line segments (resp., horizontal line segments), we have $R_v^c \neq \emptyset$ (resp., $R_h^c \neq \emptyset$). Choose any point $p \in R_v^c$ and let L_h be the maximal horizontal line segment inside R that crosses p. Since no vertical line segment in T can guard p, we conclude that no point on L_h is guarded by a vertical line segment in T. Similarly, choose any point $q \in R_h^c$ and let L_v be the maximal vertical line segment inside R that contains q. By an analogous argument, we conclude that no point on L_v is guarded by a horizontal line segment. Since L_h and L_v are maximal and have perpendicular orientations, L_h and L_v intersect inside R. Therefore, no orthogonal line segment in T can guard the intersection point of L_h and L_v, which is a contradiction. □

Given P, let $H(P)$ denote the subset of the boundary of P consisting of line segments that are immediately to the right of or below P. Let $B(P)$ denote the partition of $H(P)$ into line segments induced by the edges in $S(P)$. The following lemma follows by Lemma 1 and Observations 1 and 2:

Lemma 2. *Every orthogonal polygon P has an optimal cover $T \subseteq B(P)$.*

Observation 3. *Let P be an orthogonal polygon and consider its corresponding set $R(P)$ of rectangles induced by edges in $S(P)$. Every rectangle $R \in R(P)$ is seen by exactly one vertical line segment in $B(P)$ and exactly one horizontal line segment in $B(P)$. Furthermore, if $T \subseteq B(P)$ is a cover of P, then every rectangle in $R(P)$ must be seen by at least one horizontal or one vertical line segment in T.*

We denote the horizontal and vertical line segments in $B(P)$ that can see a rectangle $R \in R(P)$ by R_V and R_H, respectively. Observation 3 leads us to reducing the problem to the minimum-weight vertex cover problem on graphs. We construct an undirected weighted graph $G_P = (V, E)$ associated with P as follows: each line segment $s \in B(P)$ corresponds to a vertex $v_s \in V$ such that the weight of v_s is the length of s (we denote the vertex in V that corresponds to the line segment $s \in B(P)$ by v_s). Two vertices $v_s, v_{s'} \in V$ are adjacent in G_P if and only if the line segments s and s' can both see a fixed rectangle $R \in R(P)$. See Figure 5 for an illustration of the reduction. By Observation 3 the following result is straightforward:

Observation 4. *For each rectangle $R \in R(P)$, there exists exactly one edge in G_P that is correspond to R.*

We now prove the following result:

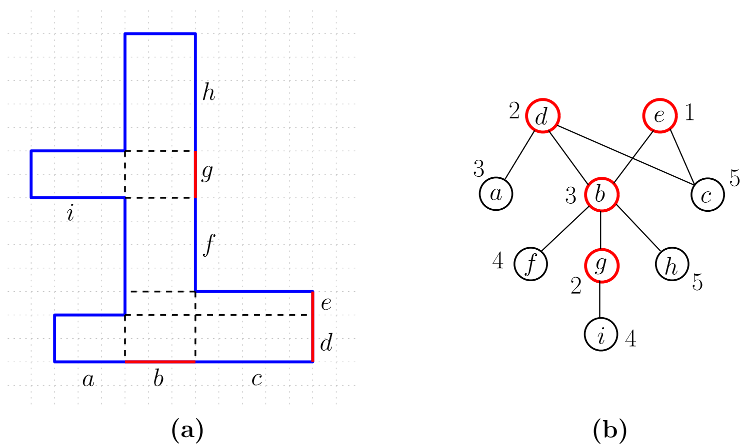


Figure 2: An illustration of the reduction; each grid cell has size 1×1. (a) An orthogonal polygon P along with the elements of $E(P) \cup S(P)$ labeled as $a, b, c, \ldots, i$. (b) The graph G_P associated with P; the integer value besides each vertex indicates the weight of the vertex. The vertices of a minimum-weight vertex cover on G_P and their corresponding guarding line segments for P are shown in red.

Theorem 1. *The MLSC problem on P reduces to the minimum-weight vertex cover problem on G_P.*

Proof. Let S_0 be a vertex cover of G_P and let C_0 be a cover of P defined in terms of S_0; the mapping from S_0 to C_0 will be defined later. Moreover, for each vertex v of G_P let $w(v)$ denote the weight of v and for each line segment $s \in C_0$ let $len(s)$ denote the length of s. We need to prove that S_0 is a minimum-weight vertex cover of G_P if and only if C_0 is an optimal cover of P. We show the following stronger statements:

- for any vertex cover S of G_P, there exists a cover C of P such that $\sum_{s \in C} len(s) = \sum_{v \in S} w(v)$, and
- for any cover C of P, there exists a vertex cover S of G_P such that $\sum_{v \in S} w(v) = \sum_{s \in C} len(s)$.

Part 1. Choose any vertex cover S of G_P. We find a cover C for P as follows: for each edge $(v_s, v_{s'}) \in E$, if $v_s \in S$ we locate a guarding line segment on the boundary of P that is aligned with the line segment $s \in B(P)$. Otherwise, we locate a guarding line segment on the boundary of P that is aligned with the line segment $s' \in B(P)$. Since at least one of v_s and $v_{s'}$ is in S, we conclude by Observation 4 that every rectangle in $R(P)$ is guarded by at least one line segment located on the boundary of P and so C is a cover of P. Moreover, for each vertex in S we locate exactly one guarding line segment on the boundary of P whose length is the same as the weight of the vertex. Therefore, $\sum_{s \in C} len(s) = \sum_{v \in S} w(v)$.

Part 2. Choose any cover C of P. We construct a vertex cover S for G_P as follows. By Observation 2, let T' be the regular cover obtained from C. Moreover, let M be the partition of T' into line segments induced by the edges in $S(P)$. By Lemma 1, for any rectangle $R \in R(P)$, there exists a set $C'_R \subseteq C$ such that all line segments in C'_R have the same orientation and collectively guard R. Therefore, M is also a cover of P. Now, let S be the subset of the vertices of G_P such that $v_s \in S$ if and only if $s \in M$. Since M is a cover of G_P we conclude, by Observation 4, that S is a vertex cover of G_P. Moreover, we observe that the total weight of the vertices in S is the same as the total length of the line segments in M and, therefore, $\sum_{v \in S} w(v) = \sum_{s \in C} len(s)$. □

We next show that the graph G_P is bipartite.

Lemma 3. *Graph G_P is bipartite.*

Proof. The proof follows from the facts that (i) we have two types of edges in G_P; those that correspond to the vertical line segments in $B(P)$ and those that correspond to the horizontal line segments in $B(P)$, and that (ii) no two vertical line segments in $B(P)$ nor any two horizontal line segments in $B(P)$ can see a fixed rectangle in $R(P)$. □

It is easy to see that the construction in the proof of Theorem 1 can be completed in polynomial time. Therefore, by Theorem 1, Lemma 3 and the fact that minimum-weight vertex cover is solvable in polynomial time on bipartite graphs [15], we have the main result of this section:

Theorem 2. *Given an orthogonal polygon P with n vertices, there exists an algorithm that finds an optimal cover of P in time polynomial in n.*

3 The MCSC Problem: NP-hardness Result

In this section, we show that the following problem is NP-hard.

MCSC WITH HOLES
Input: An orthogonal polygon P possibly with holes.
Output: An optimal solution for the MCSC problem on P.

We show NP-hardness by a reduction from the *minimum hitting of horizontal unit segments* problem, which we call it the MIN SEGMENT HITTING problem. The MIN SEGMENT HITTING problem is defined as follows.

Definition 2 (Hassin and Meggido [4], 1991)**.** *Given n pairs (a_i, b_i), $i = 1, \ldots, n$, of integers and an integer k, decide whether there exist k orthogonal lines $l_1, \ldots, l_k$ in the plane such that each line segment $[(a_i, b_i), (a_i + 1, b_i)]$ is hit by at least one of the lines.*

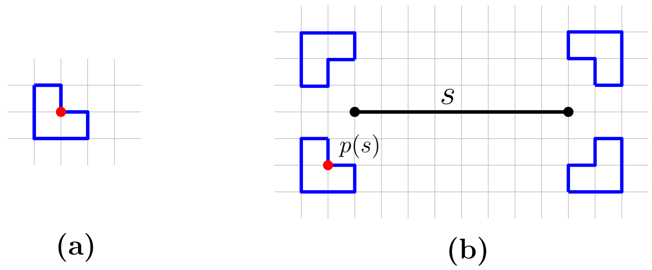


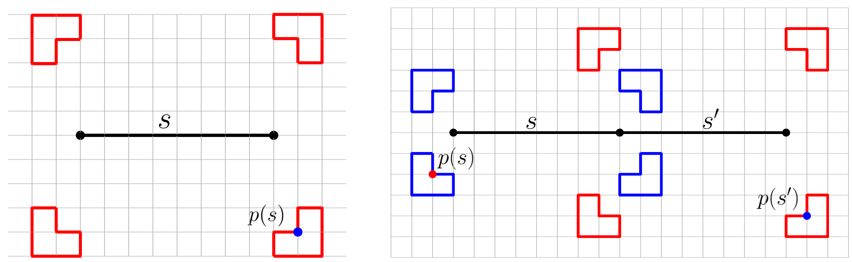
Figure 3: (a) An L-hole gadget; each grid cell has size $\frac{1}{12} \times \frac{1}{12}$. (b) The L-holes associated with a line segment $s \in I$, where the x-coordinate of a_s is even.

Hassin and Meggido [4] prove that the MIN SEGMENT HITTING problem is NP-complete. Let I be an instance of the MIN SEGMENT HITTING problem, where I is a set of n horizontal unit-length segments. We construct an orthogonal polygon P (with holes) such that there exists a set of k orthogonal lines that hit the segments in I if and only if there exists a set C of $k+1$ orthogonal line segments inside P that collectively guard P. Throughout this section, we refer to the segments in I as *unit segments* and to the segments in C as *line segments*.

Gadgets. We first observe that any two unit segments in I can share at most one point, which must be a common endpoint of the two unit segments. Let s be a unit segment in I. We denote the left and right endpoints of s by a_s and b_s, respectively. Moreover, let $N(s)$ denote the set of unit segments in I that have at least one endpoint with x-coordinate equal to that of a_s or b_s. Our reduction refers to an L-hole, which we define as a minimum-area orthogonal polygon with vertices at grid coordinates such that exactly one of which is a reflex vertex. Figure 3(a) shows an L-hole. We constrain each grid cell to have size $\frac{1}{12} \times \frac{1}{12}$. An L-hole may be rotated by $\pi/2$, π or $3\pi/2$. For each unit segment $s \in I$, we associate exactly four L-holes with s depending on the parity of the x-coordinate of a_s:

- If the x-coordinate of a_s is even, then Figure 3(b) shows the L-holes associated with s. The L-holes associated with s do not interfere with the L-holes associated with the line segments in $N(s)$ because the unit segments in $N(s)$ have the vertical distance at least one to s. Note the red vertex on the bottom left L-hole of s; we call this vertex the *visibility vertex* of s, which we denote $p(s)$.

- If the x-coordinate of a_s is odd, then Figure 4a shows the L-holes associated with s. Note that, in this case, the L-holes are located such that the vertical distance between any point on an L-hole and s is at least 3/12. By an analogous argument, we observe that the L-holes associated with s do not interfere with the L-holes associated with the line segments in $N(s)$. Note the blue vertex on the bottom right L-hole of s; we call this vertex the *visibility vertex* of s, which we denote $p(s)$.



(a) The L-holes associated with a line segment $s \in I$, where the x-coordinate of a_s is odd.

(b) An illustration of the L-holes associated with two line segments in I that share a common endpoint.

Figure 4: An illustration of the gadgets used in the reduction.

Let s and s' be two unit segments in I that share a common endpoint. Since s and s' have unit lengths the x-coordinates of a_s and $a_{s'}$ have different parities. Therefore, the L-holes associated with s and s' do not interfere with each other. Figure 4b shows an example of two unit segments s and s' and their corresponding L-holes. We now describe the reduction.

Reduction. Given an instance I of the Min Segment Hitting problem, we first associate each unit segment in $s \in I$ with four L-holes depending on whether the x-coordinate of a_s is even or odd. After adding the corresponding L-holes, we enclose I in a rectangle such that the all unit segments and the L-holes associated with them lie in its interior. Finally, we create a small rectangle on the bottom left corner of the bigger rectangle (see Figure 5) such that any orthogonal line that passes through the smaller rectangle cannot intersect any of the unit segments in I. See Figure 5 for a complete example of the reduction. Let P be the resulting orthogonal polygon. We first have the following observation.

Observation 5. *Let s be a unit segment in I. Moreover, let l be a vertical line segment contained in P that can see $p(s)$. If l does not intersect s, then $p(s')$ is not orthogonally visible to l for all $s' \in I \setminus \{s\}$.*

We now show the following result.

Lemma 4. *There exist k orthogonal lines such that each unit segment in I is hit by one of the lines if and only if there exists $k + 1$ orthogonal line segments contained in P that collectively guard P.*

Proof. ($\Rightarrow$) Suppose there exists a set S of k lines such that each unit segment in I is hit by at least one line in S. Let $L \in S$ and let $L_P = L \cap P$. If L is horizontal, then it is easy to see that L, and therefore L_P, does not cross any L-hole inside P. Similarly, if L is vertical and passes through an endpoint of some unit segment(s) in I, then neither L nor L_P passes through the interior of any

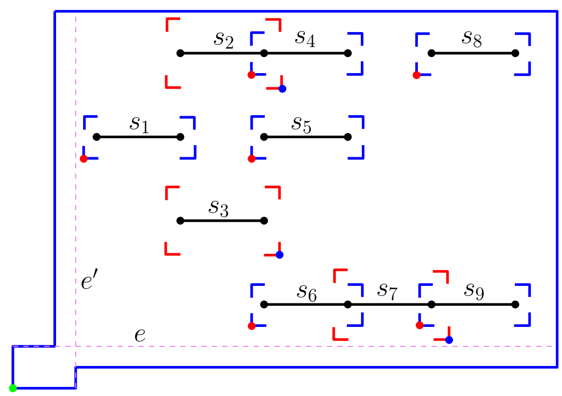


Figure 5: A complete example of the reduction, where $I = \{s_1, s_2, \ldots, s_9\}$, with the assumption that the x-coordinate of a_{s_1} is even. Each line segment that has a bend represents an L-hole associated with a unit segment. The visibility vertices of the unit segments in I are shown red or blue appropriately. Note the green vertex on the lower left corner of the smaller rectangle; this vertex is only visible to the line segments that pass through the interior of the smaller rectangle, which in turn cannot intersect any unit segment in I.

L-hole in P.[1] Now, suppose that L is vertical and passes through the interior of some unit segment $s \in I$. Translate L_P horizontally such that it passes through the midpoint of s. Since unit segments have endpoints on adjacent integer grid point, L_P still crosses the same set of unit segments of I as it did before this move. Moreover, this ensures that L_P does not cross any L-hole inside P. Consider the set $S' = \{L_P \mid L \in S\}$.

We observe that the line segments in S' cannot guard the interior of the smaller rectangle. Moreover, if the all line segments in S' are vertical or all of them are horizontal, then they cannot collectively guard the bigger rectangle entirely.[2] In order to guard P entirely, we add one more orthogonal line segment C as follows: if the all line segments in S' are vertical (resp., horizontal), then C is the maximal horizontal (resp., the maximal vertical) line segment inside P that aligns with the upper edge (resp., the right edge) of the smaller rectangle of P; see the line segment e (resp., e') in Figure 5. If the line segments in S' are a combination of vertical and horizontal line segments, then C can be either e or e'. It is easy to observe that now the line segments in S' along with C collectively guard P entirely. Therefore, we have established that the entire polygon P is guarded by $k + 1$ orthogonal line segments inside P in total.

($\Leftarrow$) Now, suppose that there exists a set M of $k+1$ orthogonal line segments contained in P that collectively guard P. Let $c \in M$ and let L_c denote the line induced by c. We find k lines that form a solution to instance I by moving the

[1] Note that it is possible that L passes through the boundary of some L-hole.

[2] Specifically, in either cases, there are regions between two L-holes associated with different unit segments that cannot be guarded by any line segment.

line segments in M accordingly such that each unit segment in I is hit by at least one of the corresponding lines. Let $c_0 \in M$ be the line segment that guards the bottom left vertex of the smaller rectangle of P. We know that L_{c_0} cannot guard $p(s)$ for all line segments $s \in I$. For each unit segment $s \in I$ in order, consider a line segment $l \in M \setminus \{c_0\}$ that guards $p(s)$. If l is horizontal and L_l does not align s, then move l accordingly up or down until it aligns s. Therefore, L_l is a line that hits s. Now, suppose that l is vertical. If l intersects s[3], then L_l also intersects s. Otherwise, by Observation 5, $p(s)$ is the only visibility vertex that is visible to l. Therefore, move l horizontally to the left or to the right until it hits s. Therefore, L_l is a line that hits s after this move.

We observe that we obtained exactly one line from each line segment in $M \setminus \{c_0\}$. Therefore, we have found k lines such that each unit segment in I is hit by at least one of the lines. This completes the proof of the lemma. □

By Lemma 4, we obtain the main result of this section:

Theorem 3. *The* MCSC WITH HOLES *is* NP*-hard.*

4 Conclusion

In this paper, we studied the problem of guarding an orthogonal polygon P with sliding cameras that was introduced by Katz and Morgenstern [8]. We considered two variants of this problem: the minimum-cardinality sliding cameras problem (in which the objective is to minimize the number of sliding cameras used to guard P) and the minimum-length sliding cameras problem (in which the objective is to minimize the total length of trajectories along which the cameras travel).

We gave a polynomial-time algorithm that solves the minimum-length sliding cameras problem exactly even for orthogonal polygons with holes, answering a question asked by Katz and Morgenstern [8]. We also showed that the minimum-cardinality sliding cameras problem is NP-hard when P contains holes, which partially answers another question asked by Katz and Morgenstern [8].

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