Defensive Strategies for a Soccer Goalkeeper with a Single Adversary

Robert Fraser¹, Alejandro Salinger¹, Reza Dorrigiv¹, Joseph D. Horton², Alejandro López-Ortiz¹
¹ David R Cheriton School of Computer Science, University of Waterloo
² Faculty of Computer Science, University of New Brunswick

The 2 player soccer problem

We are interested in determining optimal strategies for automated players in a simulated soccer environment. We start by looking at a two player version of the problem and we identify scenarios where one of the players can win. Further, we analyze the optimal strategy of a goalkeeper when facing a single opponent who has possession of the ball.

We consider the following assumptions:
• Players are points.
• Motion is of constant speed and restricted to the plane.
• Players have perfect reaction time.
• The forward can only score if his unobstructed shooting angle is greater than or equal to a predefined parameter δ.

We define the following winning conditions:
• The goalkeeper wins if he converges to the point representing the forward.
• The forward wins if he acquires the δ shooting angle.

Static regions of the plane

Given our assumptions, we can immediately identify trivial regions of the plane defined by the position of the posts and the δ angle.

δ-circle: Circular region formed by the two posts and the δ angle. It is a necessary condition that the forward is inside this region in order to have a δ shooting angle.

2δ-circle: Circular region formed by the two posts and twice the δ angle. If the forward is inside this region, he is guaranteed to have a δ shooting angle, regardless of the position of the goalkeeper.

Dynamic regions of the plane

What is the region in which the goalkeeper would win a race to the 5-circle?

This region is defined by the complement of the union of all circles centered on the 5-circle having the goalkeeper on the circumference. The boundary of this region forms a limaçon of Pascal. If the forward is outside this region, the goalkeeper can reach the forward before he gets to the 5-circle, and hence the goalkeeper wins.

What is the region in which the forward wins the race to the 25-circle?

This region is also defined by a limaçon of Pascal, this time with the circles centred along the 25-circle. If the forward is inside this region, he can reach the 25-circle before the goalkeeper, and hence wins.

What is the region in which the forward already has a shooting angle of at least δ?

This region is characterized by two circles. Each is defined by the goalkeeper, one of the posts, and the δ angle. In the same way that the δ-circle was defined. However, there are areas contained by these circles that lie outside of the δ-circle where the forward does not have a shot.

Defensive strategies

There are several plausible strategies that a goalkeeper may adopt to defend:
• Run towards the post nearest to the forward.
• Run towards the forward.
• Run towards the bisector of the angle formed by the forward and the two posts.

The first two strategies can fail in some circumstances, but the third strategy was found to be effective. Provided that the goal is on the bisector of the angle formed by the forward and the two posts, he can remain on this line regardless of the actions of the forward. Further, the goalkeeper will generally be able to move towards the forward while remaining on this line, thus converging with the forward and winning.

Proof Sketch

We wish to prove that for any points f and f’, the distance d from f to f’ is greater than or equal to the distance from any point on the bisector b to f’ to some point on the bisector b’ at f. The greatest distance that the goalkeeper would have to run is the maximum of d and the distance d’ from p to p’, which are the points of intersection of the goal line with b and b’, respectively.

Suppose that b and b’ are parallel or divergent as they approach the goal line. In this case, the maximum distance that the goalkeeper would have to run would be d’.

Now suppose that b and b’ are convergent. By definition, the shortest distance from b to b’ will be less than d, unless the goalkeeper is required to run along the goal line. Again, we are interested only in d’.

Using Maple, we proved that in this model d’ ≤ d for all values of f’ and f’. Therefore, the goalkeeper is able to remain on the bisector once he is on it. Further, he will usually be able to advance on the forward as well, this is incorporated into the strategy and shown by g’ in the figure above.

Conclusions

We have shown several characteristic regions of the plane identifying winning positions for each player in our model of a two player soccer environment.

In addition, we have identified a winning strategy for the goalkeeper, which is to remain on the bisector of the angle formed by the forward and the two posts. We have shown that the goalkeeper can remain on the bisector of the forward’s angle, no matter what the actions of the forward are.

Future work

We would like to close the gap: there remains a region in the plane where we are still unsure of the outcome of the game. These regions depend on the strategies that the players employ. Therefore, we need to identify ideal strategies for the forward, and for the goalie when he is not on the bisector.

Finally, we would like to relax the constraints in our model, such that:
• Players have dimensions,
• The ball can be kicked over the goalkeeper,
• There is some amount of reaction time,
• Physical properties are considered, such as momentum, and
• Some principles of the game are incorporated. For example, a player runs slower when in possession of the ball, or the goalie could dive.

Acknowledgements

This research was supported by the Natural Sciences and Engineering Research Council (NSERC) and the University of Waterloo.

Contact information

Robert Fraser
alejandro.salinger@uwaterrlo.ca

David R. Cheriton School of Computer Science
University of Waterloo