Comp 7720 - Online Algorithms

Assignment 0: Preparation

Shahin Kamalli
University of Manitoba - Fall 2018

Due: Saturday, September 22nd at 23:59pm

September 15, 2018

‘When I went to school (University of Manitoba), they (Instructor of the Online Algorithms course) asked me what I wanted to be when I grew up (graduate from the university). I wrote down ‘happy’. They told me I didn’t understand the assignment (the one below), and I told them (anonymously, on Piazza) they didn’t understand life ... John Lennon (almost)

Please pay attention to the followings when preparing/submitting your assignment:

• This assignment has no grade but will be looked at. You have to submit it by the mentioned deadline. Assuming you have sufficient background, it should not take you more than an hour or two to do this assignment. Failing to submit might convince the instructor that you don not have sufficient background to take this course.

• The assignment is intended to help you evaluate your basic knowledge of algorithms. As well, this assignment prepares you become comfortable with Piazza, UMLearn, and LATEX.

• If you struggle with this assignment or are not familiar with some subjects in this assignment, I strongly recommend you to reconsider taking the course and drop it if required (beware that dropping the course is OK but failing it is not).

• Submit your answers electronically using UMLearn. You should submit a single pdf file. You are encouraged to prepare your assignment using LATEX (but you don’t have to). The source files can be found on the webpage of the course.
Problem 1  Piazza Practice

Please log into Piazza and post a note or question. The note should contain your name and the content can be anything. For example, you can write your research interest, a favourite quote, a concern/suggestion for improving the class, etc. You can choose to post anonymously or under your own name (include your name in the body in case of anonymous submission). If you face any problem accessing Piazza, you should resolve it asap. Feel free to contact me about it.

Problem 2  Boring Algebra

You might look at cooking as an art which can be enjoyable and relaxing (really?). To love cooking, however, does not imply loving knives and cooking boards (you should just be OK with them). In this course, we love designing algorithms (cooking) but use algebra as our tools (knives and cooking boards) for designing and analysis of our algorithms. Let’s assess your knowledge of algebra here:

1. \(1 + 2 + 4 + \ldots + 2^n = ?\)
2. \(1 + c + c^2 + \ldots c^n = ?\)
3. \(1 + 4 + 9 + \ldots + n^2 = ?\)
4. \(1 + 8 + 27 + \ldots + n^3 = ?\)
5. Indicate the asymptotic order of \(T(n)\) where \(T(1) = 10\) and \(T(n) = 3T(n/2) + 100n\).

Problem 3  Asymptotic Notations

You are expected to be completely comfortable with asymptotic notations. Indicate whether any of the above statements are correct or not. Justify your answers.

1. If \(f(n) \in O(g(n))\) and \(g(n) \in \Theta(h(n))\) then \(f(n) \in \Theta(h(n))\).
2. \(n^2 \in \Theta(n^{2+\epsilon})\) for sufficiently small values of \(\epsilon\).
3. For any two functions \(f(n)\) and \(g(n)\), we have either \(f(n) \in o(g(n))\) or \(f(n) \in \omega(g(n))\).
4. \(f(n)^n \in \Theta(f(n)^{2n})\).

Problem 4  Data Structures

You are expected to have a solid background in data structures. Here you can assess your knowledge of data structures:

1. Indicate, with an example, the difference between an abstract data type and a data structure.
2. Indicate the time complexity of the search() operation when a dictionary data structure is implemented with I) a linked list II) a (general) binary search tree.
3. If you go to a job interview (e.g., with Google) and they ask how you to implement a dictionary data type formed by integers keys, what data structure you use?
4. Can randomization help to improve the time complexity of search() operation in a dictionary implemented by an AVL tree?

Problem 5  Randomization

Randomization often help us design better algorithms in terms of time complexity of quality of solutions.

1. The measure for studying randomized algorithms is their expected performance (e.g., expected running-time or, in our case of online algorithms, the expected cost). Indicate what the expected running-time/cost mean and what the expectation is taken over.
2. True or False: if a randomized algorithm has expected running time \(O(n)\) for an input of length \(n\), then it never runs in time more than \(O(n)\) for any input. Justify your answer.
• Assume a randomized online algorithm in which an adversary creates worst-case inputs for the algorithm. The adversary is oblivious in the sense that it knows the algorithm’s code but not the outcome of its coin flips. Indicate what the “expected worst-case cost” of the algorithm mean in such scenario.

Problem 6  Graphs
You have probably seen graphs in your previous courses. Let’s see if you remember. Indicate whether the followings are correct or not. Justify your answers. For a graph $G$, assume $n(G)$ and $m(G)$ respectively denote the number of vertices and edges of $G$.

• If $G$ is connected, then $m(G) \geq n(G) - 1$.
• The number of edges in a graph is linear to the number of its vertices, i.e., for any $G$, we have $n(G) \in \Theta(m(G))$.
• Any tree is a Hamiltonian graph.
• It is possible to color any tree of degree at most $d$ using $d + 1$ colors.