Write your name here (no need for student id, etc.) Elmer Hartman

- Do not open this booklet until instructed.
- It is an open-book exam. Your can use any printed/written material from the course. You are not allowed to use laptops/cell-phones. Please turn off your cell phones and put them in your bags.
- Manage your time. We start the exam at 9:00 and end the exam at 12:00. The exam location is E2-160 EIT Complex (seats 1-27). You have three hours.
- There are XXX pages (including this cover page). Write your answers in the provided boxes.
- In the unlikely case that you find the exam too long/hard, do not panic. Normally, the marks will be scaled so that the highest mark gets the full mark. Remark: In principle, you can collaborate in returning blank exams and I will be obliged to give you all full mark if no one writes the exam. According to the game theory, however, it is unlikely to happen. But, just in case, I added ‘normally’ to the beginning of the sentence.
- If you are not sure about the answer to a true/false question, work on other question and get back to it at the end. Do not leave true/false questions blank as there is no penalty for wrong answers. The questions are NOT ordered by their level of difficulty; indeed the first questions are likely to be harder.
- Do not waste your time writing lengthy answers. You can be succinct and yet precise. The provided space for each question is estimated to be 3 times more than the required space. Note that your time is limited.
- There are more important things in life than this exam. So, relax and smile. Also, there are more important components to this course than this exam. So, relax further and enjoy (but manage your time while relaxing).
Problem 1  True/False Questions [40 marks]

Remark: The first question includes 20 True/False questions. They cover all materials in the slides. Here are some samples. Note that the actual number of questions in this part will be more than this sample.

Indicate whether any of the the following statements is true or false. There is no need to justify your answers.

- One bit of advice is sufficient to achieve an optimal solution for the cow-path problem  True  False

- An online bidding algorithm in which bids are powers of 1.5 has a better competitive ratio than the doubling algorithm  True  False

- No randomized list-update algorithm has a competitive ratio better than that of Move-To-Front  True  False

- Consider two bin packing algorithms A and B. If competitive ratio of A is better than B then the average-case performance of B is better than A.  True  False

- Double-coverage algorithm answers the k-server conjecture in the affirmative for paths and trees  True  False

- Greedy algorithm is the optimal deterministic algorithm for online edge-coloring.  True  False

- If there is an online clustering algorithm with competitive ratio $c$ then there is a bidding algorithm with competitive ratio $c/2$.  True  False

- The following is a valid solution for fault-tolerant packing of an input sequence.  True  False

\[
\begin{array}{c|c|c}
S1 & S2 & S3 \\
\hline
d(0.1) & d(0.1) & e(0.25) \\
\hline
c(0.1) & e(0.25) & e(0.25) \\
\hline
b(0.1) & & \\
\hline
a(0.3) & a(0.3) & c(0.1) \\
\hline
& & b(0.1) \\
\end{array}
\]
Problem 2  Short Answer Questions [30 marks]

Remark: There will be 5 to 10 questions in this part, covering all slides, which require short answers. Here are some samples. Note that the actual number of questions in this part will be more than this sample.

Provide short answers to the following questions. You need to show your work but not formal justification is required. Your answers should fit in the provided boxes.

• Consider the following randomized algorithm for the ski-rental problem: with a chance of \( b - 2 \), buy the equipment at the beginning, and with a chance of \( \frac{2}{b} \) always rent and never buy. Recall that the cost of buying is \( b \) and the cost of renting is 1 per day; let \( x \) denote the number of days that the player goes skiing. Prove that the algorithm is not competitive. For that, you need to indicate what is the choice of adversary and what the costs of the algorithm and \( \text{OPT} \) are in that setting.

• Indicate the cost of the First-Fit algorithm for the following instance of server-renting problem:

\[ \sigma = \langle (0.8, 1, 9), (0.7, 2, 9), (0.5, 3, 9), (0.25, 4, 10), (0.05, 5, 9), (0.3, 6, 9), (0.2, 7, 10) \rangle \]

• Indicate the output of the Rank algorithm for the following instance of the bipartite matching problem. The number on the left show the index in the random permutation. Assume vertices on the right appear from top to bottom. Write your answers by indicating pairs of matched vertices.

![Bipartite Matching Diagram]
Problem 3  Path-Cow Problem [10 marks]
Consider the following algorithm for path-cow problem. The cow starts at the origin, moves $x = 1$ unit to the right. If the target is not found, the cow comes back to the origin and goes $y = 1$ unit to the left. If the target is not found, the cow comes back to the origin and repeats this procedure with $x = 3, 9, \ldots, 3^i, \ldots$ and $y = 3, 9, \ldots, 3^i, \ldots$ until the target is found.

a) Where does the adversary place the hole in order to harm the algorithm?

b) What is the competitive ratio of this algorithm?
Problem 4  Online Bidding Problem [10 marks]
Consider the following deterministic algorithms Alg1, Alg2, and Alg3 for the online bidding problem. Alg1 guesses are 1, 8, 64, ... \(8^i\), Alg2 guesses are 2, 16, 128, ..., \(2 \cdot 8^i\), and Alg3 guesses are 4, 32, 256, ..., \(4 \cdot 8^i\). Assume an algorithm that receives 2 bit of advice which indicate the algorithm which has smaller cost between Alg1, Alg2, and Alg3. What is the competitive ratio of the algorithm with 2 bit of advice?
Problem 5  Potential Function Method [15 marks]

Consider the Move-Counter-2 (MC2) algorithm for the list update problem. Here, each item has a counter associated with it. At the beginning, all counters are 0. After an access to an item $x$, the counter for $x$ is incremented. In case the counter becomes 2, MC2 moves $x$ to the front and resets its counter to 0.

a) Use an adversarial argument to show that competitive ratio of MC2 is at least 2.

b) Use a potential function argument to show the competitive ratio of MC2 is at most 3. You should clearly indicate what your potential function is, what the amortized cost of the algorithm for different scenario is, and provide an upper bound for the ratio between the amortized cost of the algorithm and OPT.
Problem 6  List Update & Compression [10 marks]

a) Apply the Burrows-Wheeler transform on the following string; show your work and the output.

\[ \text{kakabeka} \]

Assume $ precedes all characters when you sort rotations.

b) Assume an initial list $ \rightarrow A \rightarrow B \rightarrow C \rightarrow D$, i.e., initially $ is at index 0, A is at index 1, etc. Assume we use Move-To-Front on the above list to encode $ABADCAB$. Show what numbers will be encoded.

c) Assume an initial list $ \rightarrow X \rightarrow Y \rightarrow Z \rightarrow W$. A compressing scheme that uses Move-To-Front has encoded the following numbers for a text $T$. Show what the actual text is. The numbers are 2 0 3 1 0.

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1Kakabeka is a beautiful falls located in northwest Ontario; not too far from Winnipeg.
Problem 7  Work-Function Algorithm [10 marks]
Consider the 3-server problem on the following metric of size $m = 5$ where servers are initially located at nodes $c,d,$ and $e$.

- Assume the first request is to $a$. Indicate how the work-function algorithms serves the request. Show your work.

- Assume before serving the $t$'th request, the servers are located at nodes $b,c,$ and $d$. The values of work-function at time $t$ are computed as shown in the following table. Indicate how the algorithm serves the request at time $t$. 

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>...</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a,b,c$</td>
<td>23</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a,b,d$</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a,b,e$</td>
<td>19</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a,c,d$</td>
<td>21</td>
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<td>$a,c,e$</td>
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<td>$a,d,e$</td>
<td>21</td>
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<td>$b,c,d$</td>
<td>24</td>
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<td>$b,c,e$</td>
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<td>$b,d,e$</td>
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<tr>
<td>$c,d,e$</td>
<td>21</td>
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<td></td>
</tr>
</tbody>
</table>
Problem 8  Splay Trees [10 marks]

Apply the splay operation on the following splay tree when there is a request to node ‘15’. Show your steps.
Problem 9  Bin Packing [10 marks]

Consider instances of the bin packing problem in which all items are larger than 1/4. Use a weighting-function argument to provide an upper bound for the competitive ratio of the Harmonic Algorithm.
Problem 10  Advice [10 marks]

Consider a variant model for list-update in which paid-exchanges are forbidden, i.e., after accessing an item, the list can be modified only by using a free exchange (i.e., moving the accessed item closer to the front of the list). Show that in this model, for a sequence of $n$ requests and a list of length $m$, advice of size $O(n \log m)$ is sufficient to achieve an optimal solution. Follow the four steps to describe an algorithm with advice.
Problem 11 Adversarial Arguments [10 marks]

Consider the Most-Recently-Used (MRU) algorithm for paging which upon a fault evicts the most recently used page. Assume the cache is initially empty. Provide an adversarial sequence to show MRU is not competitive.