COMP 7720 - Online Algorithms

Online Bin Packing

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Review & Plan
Today’s plan

- Online bin packing with advice (finishing bin packing)
- Advice on giving a research talk (if we have time)
Breaking the Lower Bound

- Receive number of critical items with $O(\log n)$ bits of advice
- Consider **ReserveCritical**: 
  - At the beginning, reserve a space of size $2/3$ for critical items
  - **huge** items: open a new bin
  - **critical** items: place in a reserve space
  - **mini** item: place two of them in the same bin
  - **tiny** items: apply First-Fit to place in bins with critical or other tiny items
ReserveCritical Algorithm

- At the beginning, reserve a space of size 2/3 for critical items
  - **Huge** items: open a new bin (no other item goes there)
  - **critical** items: place in a reserve space
  - **mini** item: place two of them in the same bin (no other item goes there)
  - **tiny** items: apply FF to place in bins with critical or other tiny items

$$\sigma = \langle 0.3 \ 0.9 \ 0.6 \ 0.5 \ 0.1 \ 0.1 \ 0.56 \ 0.4 \ 0.3 \ 0.45 \ 0.8 \ 0.51 \ 0.41 \ 0.2 \ 0.1 \ 0.37 \ 0.3 \rangle$$
ReserveCritical Analysis

- ReserveCritical has c.r. of at most 1.5.
- Based on the final packing, we considered two cases:
  - Case 1: There is a bin opened by a tiny item $\rightarrow$ all bins have level at least $2/3$
  - Case 2: No bin is opened by a tiny item $\rightarrow$ using a weighting argument!
ReserveCritical algorithm

Theorem

*Competitive ratio of ReserveCritical is at most 1.5.*

- With $O(\log n)$ bits of advice, one can achieve a competitive ratio of 1.5
- Can we improve this?
RedBlue Algorithm (sketch)

- Instead of receiving the number of critical items in $O(\log n)$ bits, receive the ratio between critical and tiny bins in the final packing of ReserveCritical.

- Treat Huge and Mini items as before.

- Place a critical item in the reserved space of a critical bin; if no reserve space exists, open a new bin and declare it as critical.

- Place a tiny item in non-reserved space of critical bins (using FF).
  - If no such critical bin exists, place it in the available space of a tiny bin (using Next Fit).
  - If no suitable tiny bin exists, open a new bin.
  - Declare the new bin to be a critical or a tiny bin so that the ratio between the number of these bins becomes closer to the ratio received in advice.
RedBlue Algorithm (sketch)

- If the ratio between critical and tiny bins is encoded using $k$ bits of advice, RedBlue algorithm has a competitive ratio of at most $1.5 + \frac{15}{2^{k/2+1}}$.

**Theorem**

*With constant number of bits of advice, one can achieve a competitive ratio of (almost) 1.5.*

- Can we do better?
Power of Advice of Constant Size

- In fact, with a more complicated argument, we can show that with advice of constant size, one can achieve a competitive ratio of 1.47.
- Idea: pack items of size larger than $1/3$ separately from the rest.
  - How advice can help in packing items of size larger than $1/3$?
Power of Advice of Constant Size

- It is often useful to think of algorithms that ‘complement’ each other.
- Assume all items are larger than 1/3:
  - **Sbf**: All small items (< 1/2) are packed according to BestFit, and each large item (≥ 1/2) is placed in a new bin.
  - **Lbf**: All large items are packed according to BestFit, and each small item is placed in a new bin.

\[ \sigma = \langle 0.45 \ 0.6 \ 0.75 \ 0.34 \ 0.40 \ 0.56 \ 0.35 \ 0.55 \ 0.50 \rangle \]
Power of Advice of Constant Size

Theorem

When all items are larger than 1/3, the better algorithm among Sbf and Lbf has a competitive ratio of 1.39.

- With only one bit of advice, one can achieve a competitive ratio of 1.39 (when all items are larger than 1/3).
- Think of the two algorithms as ‘parallel algorithms’
- This algorithm is used as a subroutine for an algorithm which gets a competitive ratio of 1.47 with constant advice! (details skipped here).
Lower bound

- Advice of size $\Omega(n)$ is required to achieve an algorithm with c.r. $\leq 9/8$

- A reduction from binary guessing problem

- Consider

  $\sigma = \langle 0.5 + \epsilon, \ldots 0.5 + \epsilon, a_1, a_2, a_3 \ldots, a_{2m}, b_1, \ldots b_m \rangle$

  - $m$ green items
  - $a_1, a_2, a_3 \ldots, a_{2m}$ white items in range $(1/3, 1/2]
  - $b_1, \ldots b_m$ complements of smaller white items

- The algorithm should ‘guess’ whether each white item is among the larger half of smaller half of white items!
Lower bound

\( \langle 0.51, \ldots, 0.51, 0.42, 0.37, 0.4, 0.39, 0.38, 0.385, 0.388, 0.386, 0.63, 0.62, 0.615, 0.614 \rangle \)

- Guess if an item is among smaller or larger half of white items
  - open a new bin for smaller half of white items (in anticipation of their complements coming in the future)
  - for the larger half of white items, put them with green items
- The ‘type’ (being in smaller or larger half) of the white item cannot be revealed from knowing types of previous white items
- For any four mistakes in guessing, at least 1 extra bin is opened

![Diagram](image-url)
Lower bound

**Theorem**

*In order to achieve a competitive ratio better than 9/8, advice of linear size is required*

- This result can be improved to show that for a competitive ratio better than $4 - 2\sqrt{2} \approx 1.172$, a linear number of bits are required.
- No advice: best upper and lower bounds by [Heydrich and van Stee, 2015] and [Balogh et al., 2012].
- With $\Theta(n \log N)$ bits, one can achieve an optimal solution ($N$ is the cost of $\text{OPT}$) [Boyar et al., 2014].
- With $\Theta(\log n)$ bits, one can achieve a competitive ratio of 1.5 (better than all online algorithms) [Boyar et al., 2014].
- With linear number bits, one can achieve a competitive ratio of 4/3 [Boyar et al., 2014].
- For a competitive ratio better than 9/8, a linear number of bits are required [Boyar et al., 2014].
- With linear number bits, one can achieve a competitive ratio of 1.0 [Renault et al., 2014].
- With $k \geq 4$ bits, one can get a competitive ratio of $1.5 + \frac{15}{2^{k/2} + 1}$ [Angelopoulos et al., 2015].
- With $\Theta(1)$ bits, one can get a competitive ratio of 1.4702 [Angelopoulos et al., 2015].
- For a competitive ratio better than 7/6, a linear number of bits are required [Angelopoulos et al., 2015].
- For a competitive ratio better than $4 - 2\sqrt{2} \approx 1.172$, a linear number of bits are required [Mikkelsen, 2015].
References


Balogh, J.; Békési, J.; and Galambos, G. (2012). "New lower bounds for certain classes of bin packing algorithms".
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