Picture is from the cover of the book by Borodin and El-Yaniv. See Slide 6.
Introduction
In a Glance . . .

Online algorithms are

- Practical
- Diverse
- Fun (really!)
Online algorithms are

- Practical
- Diverse
- Fun (really!)

Let’s ‘play’ with online algorithms and enjoy
Instructor

Shahin Kamali

- joined UM in July 2017
- before that was a postdoc at MIT (2015-2017)
- did a PhD at U. Waterloo (2008-2014)
- In case you wonder, the accent is Persian; he is from Iran.
Students

Your turn to...

- Introduce yourself.
- Your research group, research interests.
- What makes you interested in this course?
Formalities
Lecture: Tuesdays and Thursdays, 10:00-11:15am
115 Armes
Formalities

Logistics

- Lecture: Tuesdays and Thursdays, 10:00-11:15am
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- Webpage:
  http://www.cs.umanitoba.ca/ kamalis/fall18/comp7720.html
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- Piazza: https://piazza.com/umanitoba.ca/fall2018/comp7720
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- **Piazza:** https://piazza.com/umanitoba.ca/fall2018/comp7720

- **Office hours:** 2:00 pm-3:00 pm, Tuesdays and 11:30-12:30
  Thursdays, in E2 586 or by appointment
A list of required reading will be provided on the course webpage.

No book is required to be purchased.

The following book is suggested as a reference:

Formalities

Grading

There will be:

- Four assignments
- A final exam
- A course project
Formalities

Grading

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Theorem

*The focus of this course is on learning, discovering, and research.*
There will be:

- Four assignments
- A final exam
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**Theorem**

*The focus of this course is on learning, discovering, and research.*

**Corollary**

*Don’t worry too much about your final mark (but worry a little bit).*
Formalities

Grading (cntd.)

- Four assignments:
  - 30 percent of the final mark
  - there will be extra marks for bonus questions.
  - submit only pdf files (preferably use \LaTeX)
  - we will use both Crowdmark
Formalities

Grading (cntd.)

- Four assignments:
  - 30 percent of the final mark
  - there will be extra marks for bonus questions.
  - submit only pdf files (preferably use \LaTeX)
  - we will use both Crowdmark
  - An additional assignment, Assignment 0, will be posted shortly.
    - It gives you a chance to assess your background and learn to work with \LaTeX, Piazza, and Crowdmark.
    - It gives you a chance to drop the course before it is too late if you lack the background.
Four assignments:
- 30 percent of the final mark
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- An additional assignment, Assignment 0, will be posted shortly.
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Final exam:
- 30 percent of the final mark
- It is a closed-book exam.
- A sample exam will be provided for practice
Formalities

Projects

- **Course project**
  - 35 percent of the final mark
  - Extra marks for outstanding projects (publishable projects)
  - Work individually or in groups of two

- **Projects involve:**
  - Proposal
  - Presentation
  - Final report (in form of a research paper)
Project topics will be suggested in the first few weeks of the class

- You can choose your own topic based on your research
- Come to office hours to talk about it!
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Project categories:

- Exploring possible solutions to an open problem
- Writing a survey paper on a current topic related to online algorithms
- Writing code to implement and compare the performance of online algorithms for a problem
Class participation:

- 5 percent of the final mark
- Don’t be shy; ask questions, answer my questions, seat in the frontline!
Formalities

Important Dates (tentative)

- September 6: the first class
- September 20: assignment 1 due
- September 27: project proposal due
- October 11: assignment 2 due
- October 18: chocolate cupcake day
- October 25: assignment 3 due
- November 15: assignment 4 due
- November 19: VW deadline
- November 20: project presentation starts
- December 6: final exam
- December 13: project final report due
Online Algorithms
Online Algorithms

Offline vs. Online Algorithms

- Traditional algorithms are ‘offline’ in the sense that they have the whole input in their hand.

- Online algorithms, in contrast, do not have/need the whole input in order to solve a problem
  - The input is a ‘sequence’ which is processed by the online algorithm piece-by-piece
  - The online algorithms often take irrevocable decisions to process the input.
The input is a set/sequence of items of various sizes

E.g., \(<9, 3, 8, 5, 1, 1, 3, 2, 4, 2, 4, 5, 5, 8, 6, 4, 5, \ldots >\).
Bin Packing Problem

- The input is a set/sequence of items of various sizes
  - E.g., \(<9, 3, 8, 5, 1, 1, 3, 2, 4, 2, 4, 5, 5, 8, 6, 4, 5, \ldots>\).

- The goal is to pack these items into a minimum number of bins of uniform capacity.

```
\begin{array}{cccccccc}
1 & 1 & 2 & 2 & 5 & 5 & 8 & 4 \\
9 & 5 & 8 & 4 & 5 & 5 & 8 & 6 \\
3 & 3 & 4 & 4 & 5 & & & \\
\end{array}
```
In the online setting:

- an algorithm receives items one by one
- when it receives an item, it has to place it in a bin without any knowledge about forthcoming items
- decisions of the algorithms are irrevocable (i.e., cannot move items between bins)
First Fit (FF) Algorithm

- Find the first bin which has enough space for the item, and place the item there
- Open a new bin if such bin does not exist
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Online Algorithms

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![Diagram showing First Fit (FF) Algorithm](image)
Online Algorithms

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![Diagram of First Fit Algorithm]

- 1st bin: 9
- 2nd bin: 5
- 3rd bin: 8
- 4th bin: 4
- 5th bin: 3
- 6th bin: 3
- 7th bin: 4
Online Algorithms

First Fit (FF) Algorithm

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Online Algorithms

Competitive Ratio

- We use the framework of competitive analysis to compare online algorithms.
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Let \( \text{OPT} \) denote the best possible offline solution.

Given a sequence \( \sigma \), \( \text{OPT} \) is an algorithm which packs items in \( \sigma \) in a minimum number of bins.
Online Algorithms

Competitive Ratio

- We use the framework of **competitive analysis** to compare online algorithms.
- Let $\text{OPT}$ denote the best possible offline solution.
  - Given a sequence $\sigma$, $\text{OPT}$ is an algorithm which packs items in $\sigma$ in a minimum number of bins.
- Competitive ratio of an algorithm $A$ is the maximum ratio between the cost of $A$ and that of $\text{OPT}$ over all sequences.

$$cr(A) \equiv \max_{\sigma} \frac{\text{cost}_A(\sigma)}{\text{cost}_{\text{OPT}}(\sigma)}$$
For First Fit, the competitive ratio is 1.7 [Johnson 1973]
Online Algorithms

Competitive Ratio of First Fit

For First Fit, the competitive ratio is 1.7 [Johnson 1973]

- The number of bins opened by FF for any sequence is at most 1.7 times that of $\text{OPT}$, i.e., $c.r. \leq 1.7$ (upper bound for FF)
- There are sequences for which the number of bins opened by FF is 1.7 times that of $\text{OPT}$, i.e., $c.r. \geq 1.7$ (lower bound for FF)
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- The best existing online algorithm has c.r. of 1.5783 [Balogh et al. 2017]
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- The best existing online algorithm has c.r. of 1.5783 [Balogh et al. 2017]

- No algorithm can be better than 1.54037-competitive (best general lower bound) [Balogh et al. 2015].
Ski-rental problem

- Assume you want to go skiing for $x$ number of days
  - In the online setting, the value of $x$ is unknown!
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- In the online setting, the value of $x$ is unknown!

You can buy the equipment for a one-time cost of $b$ or rent each day for a cost of 1 per day
Ski-rental problem

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  - In the online setting, the value of \( x \) is unknown!
- You can buy the equipment for a one-time cost of \( b \) or rent each day for a cost of 1 per day
- If we know \( x \), what is the best solution?
Assume you want to go skiing for \( x \) number of days

- In the online setting, the value of \( x \) is unknown!

You can buy the equipment for a one-time cost of \( b \) or rent each day for a cost of 1 per day

If we know \( x \), what is the best solution?

- Buy at the beginning if \( x \geq b \), otherwise, rent every day
Online strategy **break-even**: rent for the first $b - 1$ days and buy in the next day.
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What is the competitive ratio of Break-even algorithm?
Online strategy **break-even**: rent for the first $b - 1$ days and buy in the next day.

What is the competitive ratio of Break-even algorithm?

**Theorem**

*Competitive ratio is roughly 2, and it is the best for any deterministic online algorithm.*
In many occasions, a ‘doubling technique’ can be used to design and analyze online algorithms:

- The lost cow problem
- Online bidding
Potential function technique is a classic approach for analysis of online problems.

- The paging problem, Sleator-Tarjan proof, randomized paging.
Syllabus

Data structures

- Self-adjusting data structures
  - List update problem
  - Data compression
  - Self-adjusting binary trees, and dynamic optimality conjecture
Syllabus

Packing problems

- Weighting technique
  - Bin packing
  - Renting servers in the cloud
  - Online scheduling
Graph problems

- $k$-server problem
- Graph coloring
- Bipartite matching
Syllabus

**Computation geometry**

- Robot searching
- 2-dimensional bin packing
Advice complexity of online problems

- Online algorithms with Advice: what if we have partial information about future?
  - Algorithms with advice for paging, bin packing, list update

- Many project ideas here!
Alternative analysis techniques

- Competitive ratio is a **worst-case measure**
- Alternative analysis techniques are used to compare algorithms based on their typical behaviour
  - Bijective analysis
  - Relative worst-order analysis