Problem 1  Tracing Algorithms $[5+5+5 = 15$ marks$]$

Consider the input sequence $\sigma = (0.24, 0.60, 0.10, 0.28, 0.78, 0.17, 0.34, 0.88, 0.23, 0.35, 0.30, 0.21)$. In what follows, you are asked to provide final packings of a few bin packing algorithms for packing $\sigma$. Showing intermediate packings is not required.

a) Show the final packing of Best Fit for $\sigma$.

b) Show the final packing of Harmonic with parameter $k = 5$ for $\sigma$.

c) Show the final packing of Harmonic-Match for $\sigma$. Assume $k = 5$, that is, items of size $\leq 1/5$ belong to the same class.
Problem 2  Bounded Space Bin Packing [$7+5 = 12$ marks]

The Random-Best-Fit (RBF) algorithm for bin packing works as follows. The algorithm has at most three open bins at any given time. Upon arrival of an item $x$:

- If more than one bin have enough space for $x$, randomly select one of the bins that have enough space for $x$ and place $x$ in that bin.
- If only one bin has enough space for $x$, place $x$ in that bin.
- If no bin has enough space for $x$, open a new bin for $x$. If there are three bins open before placing $x$, to keep the number of three bins at most two, first close the bin with the larger level (the fuller bin).

a) Provide a sequence that shows the competitive ratio of RBF is at least 2.

b) Prove that the lower bound of part (b) is tight by showing the competitive ratio of RBF is at most 2.

Problem 3  Weighting Arguments [$7+7 = 14$ marks]

a) Consider a restricted version of bin packing where all items larger than $1/3$. Use a weighting argument to prove that the competitive ratio of any Any Fit algorithm Alg is at most 1.5.

b) Consider a restricted version of bin packing where all items smaller than $1/2$. Use a weighting argument to prove an upper bound for the competitive ratio of Harmonic with a large number of classes. You can assume the best approach for getting a bin with maximum density is greedy (as we discussed in the class). If required, you can refer to the slides to shorten your answer.

Problem 4  Fun with First Fit [$7$ marks]

Consider a restriction of bin packing where all items are smaller than $1/k$. Prove that First Fit has a competitive ratio of at most $\frac{k+1}{k}$.

Problem 5  Bin packing & average-case analysis [$6+6 = 12$ marks]

Consider an alternative algorithm for up-right matching problem which processes $\oplus$ points from top to bottom, and match each $\oplus$ point with the right-most unmatched $\oplus$ point on its right.

a) What is the equivalent of the above greedy algorithm in the bin packing instance? you should precisely describe the algorithm.

b) For an instance of $n$ points, assume the above algorithm is expected to match all points except $o(n)$ of them. What statement can we make in the context of bin packing algorithms that we know? A short explanation is sufficient.