Write your name here (no need for student id, etc.)  Elmer Hartman

- This acts just as a practice/sample for Exam 1. Do not expect to see the same or similar questions in the exam. To succeed in the exam, you should know all the material covered in the slides and assignments (the notes are uploaded to help you but it is not mandatory to go over them).

- **Do not open this booklet until instructed.** Do not write on the back side of pages; they will not be scanned. In exceptional cases that you need to do that, indicate that you have done so in this first page.

- It is an open-book exam. You can use any printed/written material from the course. You are not allowed to use laptops/cell-phones. Please **turn off your cell phones** and put them in your bags.

- Manage your time. We start the exam at 10:00 and end the exam at 11:20.

- If you are not sure about the answer to a true/false or shrot-answer question, work on other question and get back to it at the end. Do not leave true/false questions blank as there is no penalty for wrong answers. **The questions are NOT ordered by their level of difficulty; indeed the first questions are likely to be harder.**

- Do not waste your time writing lengthy answers. You can be **succinct and yet precise.** The provided space for each question is estimated to be 3 times more than the required space. Note that your time is limited.

- There are more important things in life than this exam. So, relax and smile. Also, there are more important components to this course than this exam. So, relax further and enjoy (but manage your time while relaxing).
Problem 1  True/False Questions [40 marks]

Indicate whether any of the following statements is true or false. There is no need to justify your answers.

a) One bit of advice is sufficient to achieve an optimal solution for the ski-rental problem.
   True □  False □

b) Doubling algorithm is the optimal online deterministic algorithm for the path-cow problem.
   True □  False □

c) Double-coverage algorithm answers the k-server conjecture in the affirmative for cycles.
   True □  False □

d) The k-server problem is a special instance of the paging problem.
   True □  False □

e) The following is the best randomized algorithm for the bidding problem: choose a value $X$ uniformly at random from the range $[0,1)$, and make guesses $X, Xe, Xe^2, \ldots, Xe^k$.
   True □  False □

f) No randomized paging algorithm has a competitive ratio better than that of LRU for the paging problem.
   True □  False □

g) Move-To-Front is a projective algorithm for the list update problem.
   True □  False □

h) There is an optimal lazy algorithm for the k-server problem.
   True □  False □

i) For the edge-coloring problem, 1 bit of advice is sufficient to achieve an online algorithm with a competitive ratio better than all purely-online algorithms.
   True □  False □

j) Flash-When-Full (FWF) has the best possible competitive ratio that a deterministic paging algorithm can have.
   True □  False □

k) Increasing the size of the cache for the FIFO algorithm never increases its cost.
   True □  False □

l) With advice of size $\Theta(n)$, it is possible to achieve an optimal algorithm for the paging problem for an input of $n$ requests.
   True □  False □

m) The Balance algorithm is the optimal deterministic algorithm for the k-server problem.
   True □  False □
Problem 2  Short Answer Questions [30 marks]

Provide short answers to the following questions.
You need to show your work but not formal justification is required.
Don't waste time by writing long answers.

a) Consider the double coverage algorithm (DCA) working for the 4-server problem defined on the following tree. Assume the servers are located at nodes $d, c, m,$ and $k$. Assume the next request is to $a$. Indicate the final position of servers after serving the request at $a$. You can show the positions on the picture OR write them in the space below.

b) Apply the Burrows-Wheeler transform on the string $ararat$\(^1\); show your work and the output. Assume $\$ precedes all characters when you sort rotations.

\(^1\)Ararat is a holy mountain for Armenian people situated in modern day Turkey.
Problem 3  Path-Cow Problem [10 marks]

Consider the following algorithm for path-cow problem. The cow starts at the origin, moves $x = 1$ unit to the right. If the target is not found, the cow comes back to the origin and goes $y = 1$ unit to the left. If the target is not found, the cow comes back to the origin and repeats this procedure with $x = 2, 4, \ldots, 2^i, \ldots$ and $y = 3, 9, \ldots, 3^i, \ldots$ until the target is found (see the figure below).

Indicate whether the algorithm is competitive or not. You need to show where the adversary places the target and what the cost of the algorithm and $\text{OPT}$ are in that case.

Hint: we have $1 + 3 + \ldots + 3^k = \frac{3^{k+1} - 1}{2}$.
Problem 4  Online Bidding Problem [10 marks]

Consider the following deterministic algorithms Alg1, Alg2, Alg3, and Alg4 for the online bidding problem. Alg1 guesses are 1, 16, 256, . . . , 16^i, Alg2 guesses are 2, 32, 512, . . . , 2 · 16^i, Alg3 guesses are 4, 64, 1024, . . . , 4 · 16^i, and Alg4 guesses are 8, 128, 2048, . . . , 8 · 16^i.

Assume an algorithm that receives 2 bit of advice which indicate the algorithm that has smaller cost between Alg1, Alg2, Alg3, and Alg4. What is the competitive ratio of the algorithm with 2 bit of advice? (assume the values chosen by adversary are arbitrary large).

Hint: \[1 + \frac{1}{16} + \frac{1}{256} + \ldots + \frac{1}{16^k} = \frac{1 - 1/16^{k+1}}{15/16} \approx 16/15\] for large values of \(k\).
Problem 5  List Update & Compression [10 marks]

a) Assume an initial list $\rightarrow A \rightarrow B \rightarrow C \rightarrow D$, i.e., initially $\$ is at index 0, $A$ is at index 1, etc.
Show the encoding of Move-To-Front on the above list for the input $ADAAB\$$. You need to show how the list is updated.

b) Assume an initial list $\rightarrow X \rightarrow Y \rightarrow Z \rightarrow W$. A compressing scheme that uses Timestamp has encoded the following numbers for a text $T$. Show what the actual text is. The numbers are 2 0 2 1 0. You need to show how the list is updated.
Problem 6  Potential Function Method [15 marks]

Consider the Move-Counter-3 (MC3) algorithm for the list update problem. In this algorithm, each item has a counter; at the beginning, all counters are 0. After an access to an item $x$, the counter of $x$ is incremented. If the update value of the counter is less than 3, the algorithm does nothing (the list is not changed). Otherwise (when the counter becomes 3), MC3 moves $x$ to the front of the list and resets its counter to 0.

a) Use an adversarial argument to show that competitive ratio of MC3 is at least 3. Assume the list is initially ordered as $a_1 \rightarrow a_2 \rightarrow a_3 \ldots \rightarrow a_m$ where $m$ is a large number.

b) Use a potential function argument to show the competitive ratio of MC3 is at most 4. To prove this upper bound, you should indicate what your potential function is, what the amortized cost of the algorithm for different scenarios is.
Problem 7  Work-Function Algorithm [10 marks]

Consider the 2-server problem on the following metric of size $m = 4$ where servers are initially located at nodes $a$ and $b$.

a) Assume the first request is to $c$. Indicate how the work-function algorithms serves the request. Show your work.

Hint: you have to find the values of work-function at time 1 first.

Assume before serving the $t$’th request, the servers are located at nodes $b$ and $d$. The values of the work function at time $t$ are computed as shown in the following table. Indicate how the algorithm serves the request at time $t$. Show your work.
Problem 8  Splay Trees [5 marks]

Apply the splay operation on the following splay tree when there is a request to node ‘8’. Show your steps.