Comp 7720 - Online Algorithms

Final Exam

University of Manitoba - Fall 2019 (S. Kamali)

‘There is no real ending. It is just the place where you stop the story ... ’  Frank Herbert

Write your name here (no need for student id, etc.)  Travis Henderson

• Do not open this booklet until instructed. You can use the last (blank) page if you needed more space.

• It is an open-book exam. Your can use any printed/written material from the course. You are not allowed to use laptops/cell-phones. Please turn off your cell phones and put them in your bags.

• Manage your time. We start the exam at 10:00 and end the exam at 11:20.

• If you are not sure about the answer to a true/false or short-answer question, work on other question and get back to it at the end. Do not leave true/false questions blank as there is no penalty for wrong answers. The questions are NOT ordered by their level of difficulty.

• Do not waste your time writing lengthy answers. You can be succinct and yet precise. Note that your time is limited.

• There are more important things in life than this exam. Also, there are more important components to this course than this exam. So, relax and enjoy (but manage your time while relaxing).
Problem 1  True/False Questions [24 marks]

Indicate whether any of the the following statements is true or false. There is no need to justify your answers.

There will be more (12) True/False questions. Here are some samples. Note that the questions in the exam could be quite different in this part.

a) First Fit is the best algorithm for the online bin packing problem with respect to worst-case performance.
   True [ ] False [ ]

b) In order to prove an algorithm A is the optimal online algorithm (with respect to competitive ratio) for a given algorithm, it suffices to prove the following two statements: I) for any input sequence, the ratio between the cost of A and Opt is at most c. II) there are sequences for which the cost of A is c times more than the cost of Opt.
   True [ ] False [ ]

c) Advice of size $O(n \log n)$ is sufficient to achieve an optimal matching of an $n \times n$ bipartite graph.
   True [ ] False [ ]

d) Greedy algorithm is the optimal deterministic algorithm for edge coloring.
   True [ ] False [ ]

e) The best randomized algorithm for bipartite matching has a strictly better competitive ratio than the best deterministic algorithm
   True [ ] False [ ]

f) There are sequences for which First Fit generates the optimal packing.
   True [ ] False [ ]

g) Best Fit has a better competitive ratio than Next Fit for renting servers in the cloud
   True [ ] False [ ]

h) With $\Theta(n)$ bits of advice, one can achieve optimal packing of any sequence.
   True [ ] False [ ]
Problem 2  Short Answer Questions [21 marks]

Provide short answers to the following questions. You need to show your work but not formal justification is required. Don’t waste time by writing long answers.

There will be more (7) True/False questions. Here are some samples. Note that the questions in the exam could be quite different in this part.

a) Show the final packing of ReserveCritical for the following instance of the bin packing problem. Assume the number of classes in \( k = 100 \). You do not need to show the intermediate packings.

\[ \sigma = (0.3, 0.6, 0.55, 0.25, 0.15, 0.28, 0.6) \]

b) Indicate the cost of the Move-To-Front algorithm for the following instance of server-renting problem:

\[ \sigma = [(0.8, 1, 9), (0.7, 2, 8), (0.35, 3, 5), (0.25, 4, 6), (0.05, 5, 7), (0.95, 6, 7), (0.2, 7, 9)] \]

c) Indicate the output of the Rank algorithm for the following instance of the bipartite matching problem. The number on the left show the indices in the random permutation. Assume vertices on the right appear from top to bottom. Write your answers by indicating pairs of matched vertices. No justification is required.
d) Consider the following packing for an instance of fault-tolerant bin packing. Indicate whether the packing is valid or invalid. Briefly justify your answer.

![Packing Diagram]

e) An intelligent student named Alice is doing research on an online problem $P$. She has been able to reduce the binary guessing problem into $P$ by showing that any online algorithm for $P$ has to make ‘guesses’ such that each correct guess has a cost of 2 and each wrong guess has a cost of 5 for the algorithm. What lower bound for competitive ratio of online algorithms with sublinear advice can be deduced from Alice’s reduction? A short justification is sufficient.
Problem 3  Bin Packing [10 marks]

Consider instances of the bin packing problem in which all items are larger than 1/4. Use a weighting-function argument to provide an upper bound for the competitive ratio of the Harmonic Algorithm for these instances of the problem.
Problem 4  Advice [10 marks]

Consider a variant of the bin packing problem in which all $n$ input items are either 1/3 or 2/3 or 1 (there is no item of another size). Moreover, the algorithm has bounded space with $k = 2$, that is, it cannot have more than 2 open (active) bins at any time. Use a reduction from the binary guessing problem to show that, in order to achieve a competitive ratio better than 5/4, advice of size $\Omega(n)$ bits is required.