Based on notes by S. Durocher.
Overview

- Review: Arrays and Binary Search
- Review: Linked Lists
- Stacks via arrays & linked-lists
- Queues via arrays/linked-lists)
- Doubly-linked lists

For further reading, refer to Open Data Structures Book (Chapters 2 and 3)
Data Structures vs Abstract Data Types

A **data structure** is a model for the organization of data and their storage allocation on a computer.

An **abstract data type** consists of:

1. a collection of data items, and
2. a set of allowable operations on the data items.
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In this module, We define three abstract data types: stack, queue, deque.

For each ADT we examine different possible implementations using different data structures: array, linked list, or doubly-linked list.
Insertion Sort

```java
int[] A = new int[8];
A[0] = 16;
...
for (int i = 0; i < 8; i++)
```

When the control gets to `int[] A = new int[8];`, eight consecutive memory location, each enough for storing an integer (e.g., each 4 bytes), are dedicated on the main memory to store $A$.

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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</thead>
<tbody>
<tr>
<td>16</td>
<td>42</td>
<td>0</td>
<td>-3</td>
<td>1</td>
<td>50</td>
<td>63</td>
<td>-9</td>
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Arrays tend to be easy to code.

Given an index $i$, accessing the corresponding array value $A[i]$ takes $O(1)$ time. This is done through random access, e.g., `staff[85].getLastName();`
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For `int a[10];` an array is created at the **compile time**

- The memory for it will be a part of the memory assigned for the code.
- Can you have the following?
  
  ```java
  Scanner scan = new Scanner(System.in);
  int num = scan.nextInt();
  int a[num];
  ```

For `int a[] = new int[10];` an array is created at the **run time**; the value of `num` is not needed at the compile time.
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Binary Search

- an important property of arrays is the ability to search a sorted array in $O(\log n)$ time

```java
// Returns the location of x in given array arr[l..r]
public static int binSearch(int[] A, int x) {
    return binSearchRec(A, 0, A.length - 1, x);
}

private int binSearchRec(int arr[], int lo, int hi, int x) {
    int result = -1;
    int mid = (lo + hi) / 2;
    if (lo <= hi && key == A[mid])
        result = mid;
    else if (lo <= hi && key < A[mid])
        result = binSearchHelper(A, key, lo, mid - 1);
    else if (lo <= hi)
        result = binSearchHelper(A, key, mid + 1, hi);
    return result;
}
```
binarySearch(A, 0, 16, 22);
**Binary Search**

```
binarySearch(A, 0, 16, 22); → mid = 8, A[8] < 22
```
Binary Search

\[
\begin{array}{cccccccccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\
2 & 5 & 6 & 8 & 9 & 9 & 12 & 15 & 17 & 20 & 21 & 22 & 29 & 31 & 32 & 45 \\
\end{array}
\]

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\text{binarySearch}(A, 0,16, 22); \rightarrow \text{mid} = 8, A[8] < 22 \rightarrow \\
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Binary Search

```
A    0  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15
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binarySearch(A, 0, 16, 22); → mid = 8, A[8] < 22 →
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\text{return } 11
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Time complexity of binary search

- On every step, the number of possible array cells to search decreases by half.
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In the worst case, the search terminates when the number of cells to be searched is one, i.e., $n/2^i = 1$ which gives $n = 2^i$, that is, $i = \log_2 n$. So, in the worst case, $O(\log n)$ steps are required, each taking constant time → binary search takes $O(\log n)$ in the worst-case.
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- So, in the worst case, $O(\log i)$ steps are required, each taking constant time $\rightarrow$ binary search takes $O(\log n)$ in the worst-case.
In addition to the array data structure, you have seen the linked list.
Linked List Simple Implementation

- Each item in the list will be an instance of class Node:
  - Here is version 1 for class Node. What’s wrong with it?

```java
public class Node {
    public int value;
    public Node next; // a pointer to the next node

    // constructor which gets both value and the pointer
    public Node(int newValue, Node newNext) {
        value = newValue;
        next = newNext;
    }

    // constructor which gets only the value; next is set to null
    public Node(int newValue) { this(newValue, null); }
}
```
Linked List with Information Hiding

Each item in the list will be an instance of class Node:

- A better version with information hiding

```java
public class Node {
    protected int value;
    protected Node next;

    public Node(int newValue, Node newNext) {
        value = newValue;
        next = newNext;
    }

    public Node(int newValue) { this(newValue, null); }

    public int getValue() { return value; }
    public void setValue(int newValue) { value = newValue; }
    public Node getNext() { return next; }
    public void setNext(Node newNext) { next = newNext; }
}
```
Linked List with Generic Types

Each item in the list will be an instance of class Node:

- An even better version which supports **generic types**:
- Here E is a parameter that can be an int, double, etc.

```java
public class Node<E> {
    protected E value;
    protected Node<E> next;
    public Node(E newValue, Node<E> newNext) {
        value = newValue;
        next = newNext;
    }
    public Node(E newValue) { this(newValue, null); }
    public E getValue() { return value; }
    public void setValue(E newValue) { value = newValue; }
    public Node<E> getNext() { return next; }
    public void setNext(Node<E> newNext) { next = newNext; }
}
```
A linked list is basically a pointer (named `head`) to a node

Plus a set of operations like `insert` and `delete`.

```java
public class LinkedList<E> {
    protected Node<E> head;
    public LinkedList() { head = null; }
    public void insert(E newValue) {
        head = new Node<E>(newValue, head);
    }
    public void delete() {
        if (head != null) head = head.getNext();
    }
}
```
LinkedList<Integer> myList = new LinkedList<Integer>();
Linked List example

```java
LinkedList<Integer> myList = new LinkedList<Integer>();
myList.insert(42);
```
LinkedList example

LinkedList<Integer> myList = new LinkedList<Integer>();
myList.insert(42);
myList.insert(-30);
LinkedList example

```java
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myList.insert(42);
myList.insert(-30);
myList.insert(16);
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```
## Dictionary Operations

<table>
<thead>
<tr>
<th>Operation</th>
<th>Unordered Array</th>
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</thead>
<tbody>
<tr>
<td>insert/delete at start</td>
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Stack ADT

- Stack is an abstract data type
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- Stack is an abstract data type
  - **Data**: a collection of elements

- Stack works based on the LIFO (Last In First Out) principle

- Stack Examples:
  - keeping track of function calls (e.g., recursion)
  - parsing brackets in a mathematical expression
  - a web browser’s “back” button
  - passengers boarding/exiting subway car
  - “undo” button on word processor
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Stack Example

isEmpty(); → true
Stack Example

isEmpty(); → true  push(20);
isEmpty(); → true  push(20);  push(25);
Stack Example

isEmpty(); → true  push(20);  push(25);  push(12);
Stack Example

isEmpty(); → true  push(20);  push(25);  push(12);  isEmpty();  
→ false
isEmpty(); → true  push(20);  push(25);  push(12);  isEmpty();  
→ false  pop();→ 12
Stack Example

isEmpty(); → true  push(20);  push(25);  push(12);  isEmpty();
→ false  pop();→ 12  push(60);
Stack Example

isEmpty(); → true  push(20); push(25); push(12); isEmpty(); → false  pop(); → 12  push(60); pop(); → 60
Stack Example

isEmpty(); → true  push(20); push(25); push(12); isEmpty(); → false  pop(); → 12  push(60); pop(); → 60  pop(); → 25
Stack Example

isEmpty(); → true
push(20); push(25); push(12); isEmpty();
→ false
pop(); → 12
push(60); pop(); → 60
pop(); → 25
isEmpty(); → false
Stack Example

isEmpty(); → true  push(20);  push(25);  push(12);  isEmpty(); → false  pop(); → 12  push(60);  pop(); → 60  pop(); → 25
isEmpty(); → false  pop(); → 20
Stack Example

is Empty(); → true  push(20); push(25); push(12); isEmpty();
→ false  pop(); → 12  push(60); pop(); → 60  pop(); → 25
isEmpty(); → false  pop(); → 20  isEmpty(); → true
Like any other abstract data type, a stack needs to be implemented via a data structure.

- We often use an array or a linked list to implement stacks.
Implementing a Stack Using an Array

- index points to the next empty location in the array.

The stack is empty:
Implementing a Stack Using an Array

- index points to the next empty location in the array.

```
Push(12)

index = 0

. . .
```

Implementing a Stack Using an Array

- index points to the next empty location in the array.

```
<p>| | | | | |</p>
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<th></th>
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</tr>
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<tbody>
<tr>
<td>12</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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</table>
```

Push(12)
Implementing a Stack Using an Array

- index points to the next empty location in the array.

Push(99)

```plaintext
|   |   | 12 | 99 |   |   |
```

index = 2
Implementing a Stack Using an Array

- index points to the next empty location in the array.

```
Push(30)

index = 3

12 99 30
```

Implementing a Stack Using an Array

- index points to the next empty location in the array.

Push(5)

```
12 99 30 5
```

index = 4
Implementing a Stack Using an Array

- index points to the next empty location in the array.

The stack contains four items: 12, 99, 30, 5.
index points to the next empty location in the array.

The stack contains **four items**:

```
12 99 30 5
```

We **pop** the top item:

```
12 99 30
```

```
5
```
public class Stack implements StackADT<Integer> {
    protected Integer[] data;
    protected int index;
    protected int size;

    public Stack(int newSize) {
        size = newSize;
        data = new Integer[size];
        index = 0;
        // initially empty
    }

    public void push(Integer item) {
        if (index < size)
            data[index++] = item;
    }

    public boolean isEmpty() {
        return (index == 0);
    }

    public Integer top() {
        Integer topItem = null;
        if (!isEmpty())
            topItem = data[index - 1];
        return topItem;
    }

    public Integer pop() {
        Integer topItem = top();
        if (!isEmpty())
            index--;
        return topItem;
    }
}
Stack st = new Stack(10);
Stack st = new Stack(10);
Stack st = new Stack(10);

st.push(25);
Stack code example

```java
Stack st = new Stack(10);
st.push(25);
st.push(5);
```
Stack Code Example

```java
Stack st = new Stack(10);
st.push(25);
st.push(5);
Integer x = st.pop();  // 5 is returned
```
Stack st = new Stack(10);

st.push(25);

st.push(5);

Integer x = st.pop();  // 5 is returned

st.push(8);
Stack code example:

```java
Stack st = new Stack(10);
st.push(25);
st.push(5);
Integer x = st.pop();  // 5 is returned
st.push(8);
boolean test = st.isEmpty();  // false
```
Stack st = new Stack(10);

st.push(25);

st.push(5);

Integer x = st.pop();  // 5 is returned

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x = st.pop();
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Stack st = new Stack(10);
st.push(25);
st.push(5);
Integer x = st.pop(); // 5 is returned
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boolean test = st.isEmpty(); // false
x = st.pop();
x = st.pop();
```
Stack Code Example

Stack st = new Stack(10);
st.push(25);
st.push(5);
Integer x = st.pop(); 5 is returned
st.push(8);
boolean test = st.isEmpty(); false
x = st.pop();
x = st.pop();
test = st.isEmpty(); true
Initially, the stack is empty

```
head: NULL
```

$\emptyset$
Initially, the stack is empty
Push(21)
Initially, the stack is empty
Push(21)
Push(42)
Implementing Stacks with Linked Lists

Initially, the stack is empty
Push(21)
Push(42)
Push(30)
Implementing Stacks with Linked Lists

Initially, the stack is empty
Push(21)
Push(42)
Push(30)
Push(16)
Implementing Stacks with Linked Lists

Initially, the stack is empty
Push(21)
Push(42)
Push(30)
Push(16)
pop() → 16 is returned
public class Stack<E> implements StackADT<E> {
    protected Node<E> head;
    public Stack() { head = null; }

    public boolean isEmpty() { return (head == null); }
    public void push(E item) {
        head = new Node<E>(item, head);
    }
    public E top() {
        E topItem = null;
        if (head != null) topItem = head.getValue();
        return topItem;
    }
    public E pop() {
        E topItem = top();
        if (head != null) head = head.getNext();
        return topItem;
    }
}
Implementing a Stack Using Linked List

Stack<Integer> st = new Stack<Integer>();
Implementing a Stack Using Linked List

```java
Stack<Integer> st = new Stack<Integer>();
```
Implementing a Stack Using Linked List

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st.push(25);
```

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st.push(5);

Integer x = st.pop();

st.push(8);

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x = st.pop();

x = st.pop();
```

```
head NULL

head 25 NULL

head 5 25 NULL

head 25 NULL

head 8 25 NULL

head NULL

head() 25 NULL

head() NULL
```
Implementing a Stack Using Linked List

```java
Stack<Integer> st = new Stack<Integer>();

st.push(25);

st.push(5);

Integer x = st.pop();

st.push(8);

boolean test = st.isEmpty(); (false is returned)

x = st.pop();

x = st.pop();

test = st.isEmpty(); (true is returned)
```
Arrays vs. Linked List for Stacks

- What are the time complexities of push(), pop(), and isEmpty?

In an array, you just need to increment/decrement the "index" for push/pop; for isEmpty, just look at it → all operations in constant time if the array is not full.

In a linked list, you just need to update the head pointer for push/pop (and create a new node for push); isEmpty is just checking the head pointer → all constant time.

The array size is fixed. If the number of pushed items exceed the size of the array, it becomes full. In case of pushing an item to a full array, you need to copy all elements to a larger array which takes $O(n)$ time → linked lists have a slight advantage over arrays!
Arrays vs. Linked List for Stacks

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Arrays & Linked for Stacks Summary

- Both arrays and linked lists can be used to implement stacks.
- In most cases, all operations (push, pop, top, isEmpty) take constant time.
  - In case of arrays, once the array become full, a push requires creating a larger array and copying all items from the old to the new ones.
  - This means that in the worst case, push takes $O(n)$ time.
Queue ADT

- Queue is an abstract data type
Queue ADT

Queue is an abstract data type

- **Data**: a collection of elements
Queue ADT

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Queue works based on the FIFO (First In First Out) principle
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Queue works based on the FIFO (First In First Out) principle

Queue Examples:

- telephone operator
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- Queue Examples:
  - telephone operator
  - serving customers at the cashier
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- Queue Examples:
  - telephone operator
  - serving customers at the cashier
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  - cars driving on a single-lane road

- Like Stacks, we can use either arrays or linked lists to implement queues
Implementing a Queue Using an Array

- As always, array has a fixed size
- There are two pointers, head and tail
  - Head points to the first empty location before the head of the queue
  - Tail points to the first empty location after the tail of the queue.
Implementing a Queue Using an Array

The queue is initially empty:

head = 0
tail = 1
Implementing a Queue Using an Array

The queue is initially empty:

- head = 0
- tail = 1

The queue contains three items:

- head = 0
- tail = 4

Items: 34, 7, 55
Implementing a Queue Using an Array

The queue is initially empty:

- head = 0
- tail = 1

The queue contains three items:

- head = 0
- tail = 4

We dequeue the item at the head:

- 7
- 55
- 34
- tail = 4
Implementing a Queue Using an Array

The queue is initially empty:

- Head = 0
- Tail = 1

The queue contains three items:

- Head = 0
- Tail = 4

We dequeue the item at the head:

- Head = 1
- Tail = 4

We enqueue 10:

- Head = 1
- Tail = 5
Implementing a Queue Using an Array

The queue is initially empty:

- head = 0
- tail = 1

The queue contains three items:

- head = 0
- tail = 4

We dequeue the item at the head:

- head = 1
- tail = 4

We enqueue 10:

- head = 1
- tail = 5

After three more calls to dequeue the queue is again empty:
Implementing a Queue Using an Array

- ‘head’ and ‘tail’ are cyclic
  - The location before the first index is $m - 1$ (array has size $m$)
  - The location after the last index is 0

Works like a circular array
Implementing a Queue Using an Array

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  - The location before the first index is $m - 1$ (array has size $m$)
  - The location after the last index is 0

Works like a circular array
public class Queue implements QueueADT<Integer> {
  protected Integer [] data;
  protected int head;
  protected int tail;
  protected int size;
  // size of the array
  protected int current;
  // number of items in the queue

  public Queue (int newSize) {
    size = newSize;
    data = new Integer [size];
    head = 0;
    tail = 1;
    current = 0;
  }

  public boolean isEmpty () {
    return (current == 0);
  }

  public void enqueue (Integer item) {
    if (current < size) {
      data[tail] = item;
      current++;
      tail = (tail + 1) % size
    }
  }

  public Integer dequeue () {
    Integer firstItem = null;
    if (!isEmpty ()) {
      head = (head + 1) % size
      current--;
      firstItem = data[head];
    }
    return firstItem;
  }
}
Implementing a Queue Using an Array

Queue q = new Queue(3);
Implementing a Queue Using an Array

```java
Queue q = new Queue(3);
q.enqueue(19);
```
Implementing a Queue Using an Array

```java
Queue q = new Queue(3);
q.enqueue(19);
q.enqueue(21);
```

```
  h   t
  19
  19 21
```
Implementing a Queue Using an Array

Queue q = new Queue(3);
q.enqueue(19);
q.enqueue(21);
q.enqueue(35);
Implementing a Queue Using an Array

Queue q = new Queue(3);
q.enqueue(19);
q.enqueue(21);
q.enqueue(35);
q.enqueue(43); // (full; the queue does not change)
Implementing a Queue Using an Array

```java
Queue q = new Queue(3);
q.enqueue(19);
q.enqueue(21);
q.enqueue(35);
q.enqueue(43); // (full; the queue does not change)
Integer x = q.dequeue();
```

Diagram:
```
  h  t
--- ---
  |   |
  h   t
  |   |
  h   t
  |   |
  h   t
  |   |
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Implementing a Queue Using a Linked List

- We require two pointers, head and tail that point to the first and last nodes in the list.

The queue is initially empty:

![Diagram showing head and tail pointers](image)
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We enqueue 16:
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We call dequeue:
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```java
public class Queue<E> implements QueueADT<E> {
    protected Node<E> head, tail;

    public Queue() {
        head = null;
        tail = null;
    }

    public void enqueue(E item) {
        Node<E> temp = new Node<E>(item);
        if (tail != null) tail.setNext(temp);
        tail = temp;
        if (head == null) head = tail;
    }

    public boolean isEmpty() {
        return (head == null);
    }

    public E dequeue() {
        E firstItem = null;
        if (!isEmpty()) {
            firstItem = head.getValue();
            head = head.getNext();
            if (head == null) tail = null;
        }
        return firstItem;
    }
}
```
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Arrays vs. Linked List for Queues

What are the time complexities of enqueue, dequeue, and isEmpty?

- In an array, you just need to increment/decrement two "indices" for head/tail for enqueue/dequeue and increment/decrement current; for isEmpty, check the value of current → all operations in constant time if the array is not full.

- In a linked list, you just need to update the head/tail pointers (and create a new node for enqueue); isEmpty is just checking the head pointer → all constant time.

- The array size is fixed. If the number of items exceed the size of the array, it becomes full. In case of enqueuing an item to a full array, you need to copy all elements to a larger array which takes $O(n)$ time → linked lists have a slight advantage over arrays!
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Arrays & Linked for Queues Summary

- Both arrays and linked lists can be used to implement queues.
- In most cases, all operations take constant time.
  - In case of arrays, once the array become full, an enqueue requires creating a larger array and copying all items from the old to the new ones.
  - This means that in the worst case, enqueue takes $O(n)$ time.