Write your name and student id here: 

‘What makes a river so restful to people is that it doesn’t have any doubt - it is sure to get where it is going, and it doesn’t want to go anywhere else.’ Hal Boyle

. Do not open this booklet until instructed.

• You are not allowed to use any printed/written material, laptops/cell-phones. Please turn off your cell phones and put them in your bags.

• Manage your time. We start the exam at 10:30 and end the exam at 11:25. You have 55 minutes.

• There are 6 pages (including this cover page). Write your answers in the provided boxes.

• In the unlikely case that you find the exam too long/hard, do not panic. The marks will be scaled so that the highest mark gets the full mark.
1. Short Answer (20 marks)

Provide your short answers in the provided boxes. There is no need to justify your answers.

1. True or False: $n^{1.001} \in \omega(n \log n)$

2. True or False: $\frac{n \log \log n}{\log n} \in o(n)$

3. Consider the following pseudocode:

```plaintext
foo(n)
1. i ← 1
2. prod ← 1
3. while i < min\{n, 2018\} do
4. for j = i to n do
5. prod ← prod \times j
6. i ← i \times 3
7. return prod
```

What is the worst-case running time of $foo(n)$? Express your answer using $\Theta$-notation in terms of $n$, and be as precise as possible.

4. Assume $T(1) = 5$ and $T(n) = 9T(n/3) + n^2$. Give an expression for the run-time of $T(n)$ using $\Theta$ notation. You might use Master theorem. Only the final answer is required.

5. True or false: The cost of QuickSort and QuickSelect are the same, if the same pivot-choosing algorithm is used.

6. True or false: Quick-select runs in $\Theta(n)$ in the average case.

7. True or false: Using an augmented AVL tree, it is possible to find the median of $n$ items in $o(n)$.

8. True or false: A binary search tree can have height $\Theta(n)$. 

2. AVL Trees (10 marks)

Consider the following AVL tree $T$:

1. Write in the missing balance factors in the figure above.

2. Perform operation $\text{insert}(45)$ on $T$.
   Draw the tree before and after each rotation performed (no need to show balance factors).
3. Skip Lists (10 marks)

Consider the skip-list $S$ shown below.

1. Show how $\text{Search}(S, 170)$ proceeds. More specifically show what nodes and in which order are visited. You should refer to the nodes using their keys and levels, e.g., you can say “node 104 at level 1”. The lowest level is level 0.

2. Show the skip list obtained by removing the key 287 from $S$, i.e., draw the skip list after performing $\text{Delete}(S, 287)$.
Consider the following binomial heaps. Show the resulting heap when we apply the operation extract-max(). In case of merging trees, if there were three binomial trees of the same order, merge the two 'older' trees (keep the new tree which is the product of previous merge). Show your work (intermediate steps).
5. Amortized Analysis (10 marks)

Consider a variant of dynamic arrays in which when an array becomes full, instead of doubling the size of the array, we quadruple the size of the array, i.e., we multiply it by four. This way, array sizes will be powers of 4. E.g., on the 65’th operation (insertion), the size of array is changed from 64 to 256. Use the aggregate method to find the amortized cost of each operation.