

# All Non-Isomorphic Residual (16, 24, 9, 6, 3)-Designs

G.H.J. van Rees  
Department of Computer Science  
University of Manitoba  
Winnipeg, Manitoba  
CANADA R3T 2N2

Abstract. We show that there are 1281 non-isomorphic residual (16, 24, 9, 6, 3)-designs.

## 1. Introduction.

A balanced incomplete block design (BIBD) is a pair  $(V, B)$  where  $V$  is a  $v$ -set and  $B$  is a collection of  $b$   $k$ -subsets of  $V$  called blocks such that each element of  $V$  is contained in exactly  $r$  blocks and any 2-subset of  $V$  is contained in exactly  $\lambda$  blocks. The numbers  $v, b, r, k, \lambda$  are parameters of the BIBD. Trivial necessary conditions for the existence of a BIBD  $(v, b, r, k, \lambda)$  are

$$(1) \quad vr = bk,$$

$$(2) \quad r(k - 1) = \lambda(v - 1).$$

A parameter set that satisfies (1) and (2) is said to be admissible.

Two BIBD's  $(V_1, B_1)$  and  $(V_2, B_2)$  are isomorphic if there exists a bijection  $\alpha : V_1 \rightarrow V_2$  such that  $B_1\alpha = B_2$ . Given a symmetric BIBD (one with  $v = b, r = k$ ), one obtains from it the residual design by deleting all elements of one block, and the derived design by deleting all elements of the complement of one block. The parameters of a derived design are  $(k, v - 1, k - 1, \lambda, \lambda - 1)$ , whereas the parameters of a residual design are  $(v - k, v - 1, k, k - \lambda, \lambda)$ .

Mathon and Rosa [3] have a very useful list of known bounds on the number of non-isomorphic BIBD's for admissible parameter sets. This paper will deal with the BIBD's with parameter sets  $(16, 24, 9, 6, 3)$ ,  $(9, 24, 8, 3, 2)$  and  $(25, 25, 9, 9, 3)$ .

## 2. Residuals.

Denniston [1] has listed all pairwise non-isomorphic symmetric designs (25, 25, 9, 9, 3). There are 78 such designs. His listing was used to produce a listing of all pairwise non-isomorphic residual designs (16, 24, 9, 6, 3).

The 78 designs were used as input to a program that deleted the blocks, one by one, from each design to produce  $78 \times 25 = 1,950$  residual designs.

Professor Wm. Kocay's very fast graph isomorphism program was then used to produce a canonical representation for each design. It could do about three a second on an Amdahl 580. These representations were then sorted and duplicates eliminated. Hence our theorem.

Theorem 1. The number of pairwise non-isomorphic residual (16, 24, 9, 6, 3)-BIBD's is 1281.

Professor Kocay's program also gave the order of the automorphism group of the design. I have summarized these results in Table 1.

**Number of Non-isomorphic Derived Designs (16, 24, 9, 6, 3)  
Whose Automorphism Group Equals  $|G|$**

$ G $	1	2	3	4	6	9	12	18	72
# of designs	1112	115	18	29	1	2	1	2	1

**TABLE 1**

Rather than writing down all  $1281 \times 24$  blocks, I will show in the next table which blocks need deleting from Denniston's list of symmetric designs to produce a representative of each isomorphism class for the 1281 designs.

# The Non-isomorphic Residual Designs

THE DESIGNS WHOSE GROUP ORDER EQUALS 1

DESIGN	DELETED BLOCK(S)
3	10 22
4	1 5 9 25
7	5 13
8	9 11 14 25
11	1 2 3 4 5 6 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
12	8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
13	7 8 12 13 15 16 20 25
14	15 16 18 19 20 23
15	25
17	4 6 14 15 16 18 20 25
18	2 8 10 11 12 13 20 24
19	1 2 3 4 5 6 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
20	8 9 10 11 12 14 16 17 18 19 20 22 23 24 25
21	3 5 7 10 11 15 17 22
22	14 16 19 21 23 24
23	15
25	3 7 8 16 17 20 24 25
26	2 3 15 18 20 22 23 25
27	5 6 9 14 15 16 19 25
28	1 7 10 14 15 17 21 25
29	13 15 17 22 23 24
30	1 2 3 4 5 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
31	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
32	1 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
33	1 2 3 4 5 6 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
34	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
35	1 2 3 4 5 6 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
36	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 17 18 19 20 21 22 23 24 25
37	1 2 4 10 13 18 19 21
38	8 9 10 15 16 21
39	1 2 3 4 5 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
40	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
41	1 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
42	1 2 3 4 5 6 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
43	1 5 13 23
44	1 2 3 4 5 6 7 8 9 10 12 13 14 15 16 17 18 19 20 21 22 23 24 25
45	1 2 3 4 5 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
46	1 5 9 10 16 19 20 24
47	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
48	1 2 3 4 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
49	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
50	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
51	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
52	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
53	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
54	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
55	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
56	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
57	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 17 18 19 20 21 22 23 24 25
58	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
59	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
60	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
61	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
62	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
63	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
64	1 9 12 17 19 21 22 23
65	1 4 5 6 16 17 21 23
66	1 4 15 17 20 21 24 25
67	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
68	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
69	1 3 8 10 18 19 24 25
70	4 6 7 9 13 17 18 21
71	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
72	3 6 12 16 20 22 24 25
73	2 4 8 17 19 20 21 23
74	6 7 9 10 13 14 22 24
75	2 4 9 15 21 23 24 25
76	2 3 9 11 17 19 21 23
77	3 5 9 15 20 22 23 24
78	1 8 9 16 17 18 19 23

## The Non-isomorphic Residual Designs Continued

THE DESIGNS WHOSE GROUP ORDER EQUALS 2

DESIGN	DELETED BLOCK(S)
1	25
2	25
3	14 18
5	4 11 12 13 16 17 18 23 24 25
6	1 14 19 24
7	4 19
9	8 12 13 16 17 18 23 24 25
10	5 16 25
13	1 4 6 9 18 19 23
14	5 8 9 10 11 12 13
16	12 23 24
17	2 3 8 12 13 17 22
18	1 4 9 14 18 19 23
21	1 4 6 9 14 18 19 23
22	4 7 8 9 10 11 12 13
24	7 11 23 25
25	1 4 9 14 18 19 23
26	6 7 8 9 10 12 13
27	2 3 7 8 12 13 17 22
36	16
43	2
46	3 7 8 12 17 21 22
48	5

THE DESIGNS WHOSE GROUP ORDER EQUALS 3

DESIGN	DELETED BLOCK(S)
15	1
19	7
23	1
28	6
37	6
38	5
64	3
65	3
66	22
69	2
70	24
72	17
73	9
74	17
75	14
76	12
77	10
78	7

THE DESIGNS WHOSE GROUP ORDER EQUALS 4

DESIGN	DELETED BLOCK(S)
1	17
2	3
4	2 3 8 13 17 22
6	2 3 13
8	2 7 8 12 13 17 21 22
10	2 12 22
15	2 7 21
17	21
22	6
23	8 12

## The Non-isomorphic Residual Designs Continued

THE DESIGNS WHOSE GROUP ORDER EQUALS 6	
DESIGN	DELETED BLOCK(S)
7	16
THE DESIGNS WHOSE GROUP ORDER EQUALS 9	
DESIGN	DELETED BLOCK(S)
7	2
29	7
THE DESIGNS WHOSE GROUP ORDER EQUALS 12	
DESIGN	DELETED BLOCK(S)
2	2
THE DESIGNS WHOSE GROUP ORDER EQUALS 18	
DESIGN	DELETED BLOCK(S)
3	1
7	1
THE DESIGNS WHOSE GROUP ORDER EQUALS 72	
DESIGN	DELETED BLOCK(S)
6	21

Also, since many people are interested in how residual designs are combined to produce symmetric designs, the next table shows into which class each of the 1,950 designs fall. The symbol  $n$  stands for the order of the group while class indicates into which isomorphism class each design falls. The design can be produced by finding the design listed in Denniston's list and deleting the block listed on the left-hand side under BLK.

The Residual Designs

BLK	DESIGN 1		DESIGN 2		DESIGN 3		DESIGN 4		DESIGN 5		DESIGN 6		DESIGN 7	
	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N
1	1128	6	1128	6	1247	18	1065	1	1006	2	1069	2	1246	18
2	1253	12	1253	12	1252	9	645	4	1006	2	1263	4	1252	9
3	1241	4	1243	4	1140	1	682	4	1114	2	1265	4	1134	1
4	1123	2	1120	2	1178	2	1000	1	1114	2	1124	2	1185	2
5	1123	2	1120	2	1131	1	1144	1	1264	72	1008	2	1139	1
6	1128	6	1128	6	1246	18	1065	1	1118	2	1069	2	1247	18
7	1253	12	1253	12	1252	9	1245	4	1118	2	1263	4	1252	9
8	1241	4	1243	4	1140	1	613	4	1113	2	982	4	1134	1
9	1123	2	1120	2	1182	2	999	1	1125	2	1127	2	1175	2
10	1123	2	1120	2	1131	1	1000	1	1075	2	1124	2	1139	1
11	1128	6	1128	6	1128	6	1065	1	1113	2	1069	2	1128	6
12	1241	4	1243	4	1140	1	593	4	1075	2	1265	4	1134	1
13	1241	4	1243	4	1140	1	1255	4	1125	2	1242	4	1134	1
14	1123	2	1120	2	1182	2	1144	1	1099	2	1008	2	1175	2
15	1123	2	1120	2	1131	1	999	1	1116	2	1127	2	1139	1
16	1128	6	1128	6	1128	6	1065	1	1101	2	1069	2	1128	6
17	1241	4	1243	4	1140	1	1261	4	1099	2	1242	4	1134	1
18	1123	2	1120	2	1178	2	999	1	1116	2	1127	2	1185	2
19	1123	2	1120	2	1182	2	1000	1	1101	2	1124	2	1175	2
20	1123	2	1120	2	1131	1	1144	1	1092	2	1008	2	1139	1
21	1264	72	1264	72	1253	12	589	4	1122	2	1264	72	1253	12
22	1241	4	1243	4	1140	1	1257	4	1089	2	982	4	1134	1
23	1123	2	1120	2	1178	2	1144	1	1092	2	1008	2	1185	2
24	1123	2	1120	2	1131	1	999	1	1122	2	1127	2	1139	1
25	1123	2	1120	2	1131	1	1000	1	1089	2	1124	2	1139	1

BLK	DESIGN 8		DESIGN 9		DESIGN 10		DESIGN 11		DESIGN 12		DESIGN 13		DESIGN 14	
	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N
1	1064	1	1008	2	1073	2	1079	1	1238	1	1067	2	1238	1
2	615	4	1008	2	1266	4	598	1	1006	2	649	1	1008	2
3	589	4	1114	2	1262	4	287	1	1238	1	884	1	1238	1
4	1007	1	1114	2	1081	2	1012	1	1114	2	1179	2	1008	2
5	1001	1	1264	72	1126	2	923	1	1248	2	964	1	1248	2
6	1064	1	1118	2	1073	2	627	1	1118	2	633	2	982	4
7	1245	4	1118	2	1266	4	1259	3	1231	2	649	1	1231	2
8	1256	4	1115	2	1244	4	614	1	1108	1	283	1	1177	2
9	1145	1	1070	2	1006	2	667	1	1104	1	1161	2	1202	2
10	1007	1	1094	2	1081	2	899	1	1077	1	882	1	1196	2
11	1064	1	1115	2	1073	2	935	1	1112	1	630	1	1183	2
12	650	4	1094	2	1262	4	594	1	1076	1	884	1	1194	2
13	1258	4	1070	2	982	4	674	1	1171	1	683	1	1200	2
14	1001	1	1106	2	1126	2	1051	1	1083	1	1021	2	1037	1
15	1145	1	1098	2	1006	2	321	1	1110	1	308	1	1040	1
16	1064	1	1103	2	1073	2	628	1	1167	1	630	1	1102	1
17	983	4	1106	2	982	4	263	1	1174	1	683	1	1042	1
18	1145	1	1098	2	1006	2	978	1	1107	1	977	2	1045	1
19	1007	1	1103	2	1081	2	671	1	1084	1	1063	2	1159	1
20	1001	1	1093	2	1126	2	967	1	1091	1	964	1	1037	1
21	646	4	1121	2	1264	72	590	1	1138	1	1245	4	1040	1
22	1239	4	1088	2	1244	4	579	1	1086	1	283	1	1102	1
23	1001	1	1093	2	1126	2	552	1	1129	1	556	2	1042	1
24	1145	1	1121	2	1006	2	290	1	1132	1	308	1	1045	1
25	1007	1	1088	2	1081	2	576	1	1135	1	882	1	1159	1

BLK	DESIGN 15		DESIGN 16		DESIGN 17		DESIGN 18		DESIGN 19		DESIGN 20		DESIGN 21	
	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N
1	1066	3	973	2	946	1	1009	2	1078	1	1238	1	1068	2
2	1260	4	973	2	652	2	1142	1	616	1	1008	2	597	1
3	644	4	973	2	1164	2	1035	1	288	1	1238	1	692	1
4	998	1	973	2	694	1	1188	2	1010	1	1114	2	979	2
5	998	1	1264	72	937	1	618	1	965	1	1248	2	880	1
6	1066	3	973	2	946	1	1022	2	631	1	1118	2	629	2
7	644	4	973	2	1207	2	1142	1	1259	3	1231	2	597	1
8	593	4	809	2	985	2	1002	1	596	1	1117	1	885	1
9	998	1	1119	2	929	1	561	2	1053	1	1071	1	1029	2
10	998	1	821	2	1004	1	1033	1	672	1	1172	1	309	1
11	1066	3	809	2	306	1	1229	1	934	1	1111	1	626	1
12	593	4	821	2	1154	2	1035	1	651	1	1096	1	692	1
13	1260	4	1119	2	1180	2	1041	1	580	1	1072	1	282	1
14	998	1	821	2	1226	1	1268	2	673	1	1105	1	1160	2
15	998	1	809	2	997	1	281	1	289	1	1173	1	922	1
16	1066	3	1119	2	306	1	1229	1	632	1	1085	1	626	1
17	1260	4	821	2	1205	2	1041	1	264	1	1109	1	282	1
18	998	1	809	2	929	1	1190	2	669	1	1082	1	562	2
19	998	1	1119	2	694	1	1197	2	1013	1	1168	1	1022	2
20	998	1	809	2	937	1	618	1	966	1	1090	1	880	1
21	593	4	1119	2	589	4	593	4	647	1	1137	1	1245	4
22	644	4	821	2	198	2	1002	1	900	1	1087	1	885	1
23	998	1	809	2	1226	1	1250	2	553	1	1136	1	1186	2
24	998	1	1119	2	997	1	281	1	320	1	1133	1	922	1
25	998	1	821	2	1004	1	1033	1	575	1	1130	1	309	1

BLK	DESIGN 22		DESIGN 23		DESIGN 24		DESIGN 25		DESIGN 26		DESIGN 27		DESIGN 28	
	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N
1	1238	1	1074	3	973	2	653	2	781	1	947	1	718	1
2	1006	2	1240	4	973	2	955	1	806	1	1032	2	322	1
3	1238	1	646	4	973	2	307	1	781	1	625	2	327	1
4	1006	2	1005	1	973	2	1189	2	806	1	601	1	1014	1
5	1248	2	1005	1	1264	72	926	1	118	2	1034	1	1014	1
6	982	4	1074	3	973	2	1021	2	988	2	947	1	717	3
7	1231	2	984	4	973	2	955	1	813	2	1207	2	327	1
8	1203	2	1240	4	819	2	996	1	555	2	980	2	328	1
9	1184	2	1005	1	811	2	1153	2	1269	2	902	1	664	1
10	1195	2	1005	1	1095	2	989	1	199	2	624	1	970	1
11	1201	2	1074	3	819	2	1225	1	589	4	1036	1	718	1
12	1193	2	984	4	1095	2	307	1	1187	2	1152	2	328	1
13	1176	2	984	4	811	2	1003	1	1198	2	187	2	322	1
14	1158	1	1005	1	819	2	1204	2	773	1	608	1	664	1
15	1043	1	1005	1	811	2	698	1	958	1	903	1	1014	1
16	1044	1	1074	3	1095	2	1225	1	280	1	1036	1	718	1
17	1080	1	646	4	819	2	1003	1	1141	1	1206	2	322	1
18	1038	1	1005	1	811	2	1249	2	522	1	902	1	970	1
19	1039	1	1005	1	1095	2	986	2	948	1	601	1	664	1
20	1158	1	1005	1	1095	2	926	1	773	1	1034	1	970	1
21	1043	1	646	4	819	2	593	4	958	1	646	4	328	1
22	1044	1	1240	4	811	2	996	1	280	1	1192	2	327	1
23	1080	1	1005	1	1095	2	1181	2	1141	1	608	1	574	1
24	1038	1	1005	1	819	2	698	1	522	1	903	1	574	1
25	1039	1	1005	1	811	2	989	1	948	1	624	1	574	1

BLK	DESIGN 29		DESIGN 30		DESIGN 31		DESIGN 32		DESIGN 33		DESIGN 34		DESIGN 35	
	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N
1	1236	3	932	1	969	1	695	1	312	1	343	1	339	1
2	973	2	477	1	51	1	1207	2	622	1	20	1	910	1
3	1236	3	304	1	787	1	940	1	840	1	396	1	829	1
4	973	2	209	1	50	1	862	1	11	1	334	1	22	1
5	1236	3	933	1	201	1	684	1	569	1	581	1	14	1
6	973	2	1022	2	139	1	586	1	635	1	548	1	600	1
7	1267	9	839	1	1281	1	1031	1	1237	1	831	1	1237	1
8	807	1	994	1	568	1	826	1	420	1	25	1	592	1
9	1100	1	486	1	837	1	1025	1	700	1	428	1	1	1
10	822	1	876	1	84	1	883	1	169	1	266	1	611	1
11	810	1	1155	1	400	1	654	1	405	1	1237	1	277	1
12	818	1	764	1	1271	1	330	1	702	1	870	1	619	1
13	1169	1	1227	1	660	1	487	1	427	1	930	1	846	1
14	818	1	1221	1	801	1	892	1	917	1	657	1	804	1
15	810	1	893	1	272	1	464	1	543	1	234	1	8	1
16	1169	1	875	1	446	1	639	1	1016	1	638	1	28	1
17	822	1	1046	1	202	1	1214	1	668	1	10	1	426	1
18	807	1	1208	1	1052	1	348	1	1156	1	296	1	31	1
19	1100	1	1215	1	802	1	865	1	1018	1	315	1	404	1
20	810	1	953	1	231	1	310	1	32	1	920	1	401	1
21	1169	1	676	1	505	1	247	1	501	1	805	1	390	1
22	818	1	927	1	273	1	496	1	302	1	409	1	394	1
23	807	1	1219	1	373	1	921	1	294	1	962	1	301	1
24	1100	1	158	1	507	1	286	1	270	1	588	1	961	1
25	822	1	945	1	116	1	881	1	6	1	291	1	578	1

BLK	DESIGN 36		DESIGN 37		DESIGN 38		DESIGN 39		DESIGN 40		DESIGN 41		DESIGN 42	
	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N
1	237	1	716	1	1236	3	690	1	771	1	599	1	909	1
2	465	1	326	1	973	2	479	1	59	1	1207	2	620	1
3	918	1	323	1	1236	3	172	1	912	1	331	1	13	1
4	1143	1	1011	1	973	2	1210	1	56	1	558	1	621	1
5	497	1	666	1	1236	3	847	1	119	1	925	1	570	1
6	337	1	719	3	973	2	1021	2	959	1	679	1	913	1
7	1232	1	329	1	1267	9	1058	1	777	1	828	1	1237	1
8	878	1	326	1	820	1	856	1	111	1	1151	1	438	1
9	60	1	1015	1	808	1	1222	1	1279	1	898	1	687	1
10	1163	1	573	1	1170	1	952	1	444	1	790	1	156	1
11	386	1	716	1	817	1	1251	1	565	1	591	1	411	1
12	133	1	329	1	1097	1	305	1	508	1	64	1	703	1
13	557	1	329	1	812	1	928	1	789	1	495	1	421	1
14	736	1	1015	1	817	1	1213	1	503	1	127	1	336	1
15	55	1	573	1	812	1	699	1	147	1	462	1	540	1
16	1021	2	716	1	1097	1	689	1	271	1	338	1	1019	1
17	957	1	323	1	820	1	995	1	836	1	827	1	1049	1
18	784	1	666	1	808	1	1220	1	380	1	397	1	670	1
19	173	1	1015	1	1170	1	489	1	114	1	864	1	845	1
20	1062	1	1011	1	1097	1	852	1	274	1	285	1	293	1
21	1234	1	323	1	817	1	697	1	976	1	637	1	279	1
22	854	1	326	1	812	1	763	1	504	1	488	1	7	1
23	851	1	573	1	1170	1	485	1	915	1	924	1	33	1
24	914	1	1011	1	820	1	661	1	357	1	311	1	482	1
25	494	1	666	1	808	1	894	1	82	1	62	1	303	1



BLK	DESIGN 43		DESIGN 44		DESIGN 45		DESIGN 46		DESIGN 47		DESIGN 48		DESIGN 49	
	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N
1	1238	1	340	1	956	1	603	1	398	1	476	1	750	1
2	1118	2	23	1	148	1	646	4	109	1	770	1	472	1
3	1111	1	610	1	200	1	188	2	521	1	782	1	66	1
4	1085	1	314	1	49	1	816	1	643	1	463	1	74	1
5	1137	1	577	1	919	1	949	1	726	1	118	2	747	1
6	1008	2	585	1	1022	2	520	1	324	1	492	1	688	1
7	1231	2	830	1	1233	1	824	2	526	1	814	1	680	1
8	1096	1	9	1	1148	1	1191	2	724	1	1030	1	44	1
9	1109	1	963	1	1162	1	905	1	435	1	1217	1	456	1
10	1087	1	292	1	52	1	816	1	524	1	101	1	1149	1
11	1238	1	1237	1	249	1	603	1	502	1	236	1	85	1
12	1117	1	395	1	141	1	981	2	313	1	1216	1	61	1
13	1072	1	408	1	872	1	118	2	189	1	491	1	268	1
14	1082	1	297	1	498	1	972	1	559	1	774	1	1273	1
15	1136	1	253	1	745	1	905	1	39	1	863	1	541	1
16	1114	2	243	1	344	1	520	1	269	1	388	1	537	1
17	1071	1	29	1	874	1	987	2	1017	1	607	1	1157	1
18	1105	1	871	1	210	1	779	1	412	1	240	1	178	1
19	1168	1	430	1	785	1	775	1	536	1	602	1	609	1
20	1133	1	677	1	944	1	972	1	640	1	772	1	723	1
21	1248	2	641	1	1235	1	623	2	572	1	584	1	416	1
22	1172	1	1150	1	159	1	1199	2	665	1	466	1	216	1
23	1173	1	429	1	1028	1	949	1	260	1	908	1	675	1
24	1090	1	849	1	284	1	779	1	241	1	904	1	155	1
25	1130	1	265	1	843	1	775	1	712	1	478	1	954	1

BLK	DESIGN 50		DESIGN 51		DESIGN 52		DESIGN 53		DESIGN 54		DESIGN 55		DESIGN 56	
	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N
1	360	1	546	1	842	1	402	1	356	1	163	1	636	1
2	19	1	211	1	943	1	3	1	887	1	886	1	149	1
3	391	1	218	1	612	1	34	1	1224	1	246	1	475	1
4	206	1	79	1	208	1	441	1	877	1	855	1	769	1
5	67	1	1024	1	184	1	733	1	890	1	860	1	48	1
6	549	1	98	1	566	1	422	1	123	1	167	1	481	1
7	662	1	1280	1	453	1	1055	1	122	1	554	1	823	1
8	24	1	183	1	500	1	1054	1	89	1	1023	1	381	1
9	361	1	125	1	721	1	389	1	1050	1	170	1	990	1
10	191	1	230	1	532	1	445	1	349	1	458	1	493	1
11	706	1	399	1	1027	1	161	1	783	1	938	1	595	1
12	77	1	1270	1	203	1	393	1	551	1	131	1	780	1
13	737	1	873	1	1277	1	300	1	993	1	896	1	776	1
14	658	1	800	1	563	1	16	1	796	1	1223	1	906	1
15	254	1	275	1	232	1	437	1	239	1	372	1	490	1
16	207	1	351	1	448	1	406	1	741	1	1026	1	118	2
17	18	1	233	1	744	1	318	1	370	1	916	1	708	1
18	298	1	869	1	866	1	678	1	765	1	799	1	815	1
19	317	1	96	1	509	1	235	1	794	1	365	1	461	1
20	950	1	730	1	70	1	759	1	346	1	788	1	792	1
21	358	1	471	1	46	1	197	1	374	1	379	1	605	1
22	415	1	473	1	105	1	12	1	1048	1	850	1	143	1
23	757	1	42	1	342	1	26	1	128	1	132	1	47	1
24	190	1	511	1	834	1	261	1	1230	1	1061	1	102	1
25	528	1	112	1	433	1	117	1	212	1	145	1	825	1

BLK	DESIGN 57		DESIGN 58		DESIGN 59		DESIGN 60		DESIGN 61		DESIGN 62		DESIGN 63	
	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N
1	714	1	746	1	227	1	907	1	939	1	150	1	219	1
2	701	1	186	1	58	1	648	1	455	1	21	1	90	1
3	715	1	174	1	468	1	175	1	685	1	205	1	1047	1
4	713	1	560	1	53	1	1211	1	110	1	316	1	220	1
5	419	1	751	1	97	1	250	1	454	1	160	1	107	1
6	325	1	567	1	385	1	550	1	942	1	691	1	971	1
7	54	1	544	1	778	1	450	1	720	1	108	1	729	1
8	539	1	527	1	606	1	861	1	564	1	17	1	94	1
9	571	1	245	1	1272	1	941	1	1278	1	1057	1	1276	1
10	642	1	531	1	78	1	129	1	72	1	738	1	583	1
11	113	1	87	1	88	1	879	1	533	1	1254	1	182	1
12	154	1	341	1	510	1	791	1	517	1	392	1	512	1
13	424	1	215	1	86	1	138	1	848	1	414	1	181	1
14	38	1	36	1	499	1	991	1	838	1	179	1	350	1
15	634	1	459	1	106	1	168	1	1147	1	1056	1	223	1
16	483	1	534	1	754	1	728	1	797	1	166	1	434	1
17	377	1	1209	1	841	1	895	1	803	1	27	1	93	1
18	134	1	103	1	104	1	889	1	753	1	758	1	224	1
19	901	1	431	1	40	1	335	1	755	1	364	1	515	1
20	1020	1	180	1	276	1	162	1	432	1	262	1	474	1
21	57	1	418	1	756	1	447	1	333	1	359	1	743	1
22	748	1	252	1	387	1	762	1	535	1	722	1	693	1
23	226	1	525	1	868	1	742	1	867	1	362	1	681	1
24	960	1	332	1	222	1	936	1	157	1	542	1	43	1
25	278	1	523	1	217	1	749	1	73	1	193	1	83	1

BLK	DESIGN 64		DESIGN 65		DESIGN 66		DESIGN 67		DESIGN 68		DESIGN 69		DESIGN 70	
	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N
1	355	1	443	1	888	1	1228	1	140	1	968	1	225	1
2	355	1	407	1	171	1	146	1	81	1	513	3	177	1
3	617	3	442	3	1166	1	410	1	353	1	153	1	376	1
4	857	1	735	1	130	1	709	1	547	1	100	1	204	1
5	1212	1	931	1	171	1	436	1	214	1	194	1	177	1
6	696	1	403	1	469	1	440	1	833	1	100	1	460	1
7	1060	1	506	1	120	1	248	1	761	1	766	1	177	1
8	1212	1	257	1	371	1	707	1	92	1	766	1	460	1
9	1212	1	257	1	130	1	727	1	739	1	153	1	152	1
10	897	1	115	1	130	1	15	1	1218	1	255	1	91	1
11	853	1	407	1	371	1	35	1	238	1	258	1	460	1
12	897	1	735	1	120	1	192	1	1165	1	194	1	91	1
13	1059	1	931	1	469	1	423	1	798	1	255	1	185	1
14	355	1	443	1	1166	1	242	1	891	1	258	1	376	1
15	853	1	931	1	795	1	4	1	366	1	153	1	225	1
16	1059	1	506	1	795	1	196	1	369	1	968	1	185	1
17	1060	1	407	1	120	1	725	1	587	1	295	1	225	1
18	696	1	403	1	795	1	1146	1	126	1	258	1	376	1
19	696	1	506	1	888	1	68	1	859	1	295	1	152	1
20	857	1	115	1	171	1	413	1	378	1	255	1	152	1
21	857	1	115	1	469	1	76	1	367	1	968	1	91	1
22	853	1	443	1	1274	3	164	1	124	1	766	1	185	1
23	1059	1	257	1	888	1	45	1	375	1	295	1	204	1
24	1060	1	403	1	1166	1	299	1	347	1	100	1	951	3
25	897	1	735	1	371	1	319	1	229	1	194	1	204	1

BLK	DESIGN 71			DESIGN 72			DESIGN 73			DESIGN 74			DESIGN 75			DESIGN 76			DESIGN 77		
	CLASS	N		CLASS	N		CLASS	N		CLASS	N		CLASS	N		CLASS	N		CLASS	N	
1	65	1		835	1		451	1		71	1		213	1		480	1		5	1	
2	663	1		69	1		467	1		71	1		731	1		151	1		439	1	
3	267	1		135	1		176	1		858	1		137	1		195	1		832	1	
4	786	1		752	1		142	1		425	1		137	1		659	1		767	1	
5	452	1		734	1		142	1		530	1		352	1		480	1		439	1	
6	41	1		752	1		545	1		417	1		80	1		659	1		354	1	
7	519	1		734	1		760	1		530	1		793	1		363	1		121	1	
8	382	1		835	1		760	1		256	1		80	1		518	1		121	1	
9	656	1		69	1		975	3		71	1		974	1		518	1		121	1	
10	384	1		538	1		142	1		768	1		137	1		518	1		1275	3	
11	63	1		514	1		467	1		425	1		793	1		582	1		30	1	
12	484	1		538	1		176	1		768	1		352	1		516	3		832	1	
13	228	1		514	1		368	1		425	1		974	1		151	1		439	1	
14	844	1		135	1		368	1		858	1		992	3		363	1		767	1	
15	457	1		259	1		467	1		768	1		80	1		195	1		5	1	
16	95	1		69	1		451	1		256	1		731	1		582	1		2	1	
17	710	1		911	3		368	1		705	3		974	1		363	1		2	1	
18	704	1		538	1		545	1		37	1		731	1		165	1		30	1	
19	251	1		259	1		545	1		37	1		213	1		480	1		5	1	
20	244	1		734	1		740	1		858	1		732	1		151	1		30	1	
21	604	1		752	1		451	1		417	1		213	1		165	1		354	1	
22	144	1		259	1		740	1		37	1		732	1		582	1		2	1	
23	221	1		135	1		176	1		530	1		352	1		659	1		767	1	
24	99	1		835	1		760	1		256	1		793	1		195	1		354	1	
25	529	1		514	1		740	1		417	1		732	1		165	1		832	1	

DESIGN 78		
BLK	CLASS	N
1	686	1
2	655	1
3	136	1
4	449	1
5	136	1
6	655	1
7	711	3
8	449	1
9	470	1
10	383	1
11	686	1
12	470	1
13	383	1
14	75	1
15	345	1
16	655	1
17	75	1
18	136	1
19	345	1
20	470	1
21	686	1
22	345	1
23	383	1
24	449	1
25	75	1

### 3. Derived Designs.

Denniston [1] states that there are six non-isomorphic (9, 24, 8, 3, 2)-BIBD's which are not derived from the (25, 25, 9, 9, 3)-BIBD. However, he didn't indicate which ones they were. Mathon [3] indicates it would be of interest to know which six of the (9, 24, 8, 3, 2)-designs, listed in Mathon and Rosa [4], are not derived. We refer to the numbering system used by Morgan [5] and extended by Mathon and Rosa [4].

Theorem 2. Designs 13, 21, 22, 34, 35 and 36 are not derived.

Proof. Each of these designs contain four blocks isomorphic to

$$\begin{aligned} B_1 &= \{1, 2, 3\} \\ B_2 &= \{1, 2, 4\} \\ B_3 &= \{1, 3, 4\} \\ B_4 &= \{2, 3, 4\}. \end{aligned}$$

If a (9, 24, 8, 3, 2)-BIBD containing  $B_1, B_2, B_3$  and  $B_4$  based on  $V = \{1, 2, \dots, 9\}$  is imbedded in a (25, 25, 9, 9, 3)-BIBD then there must be at least 18 other varieties in the extensions of  $B_1, B_2, B_3$  and  $B_4$  to ensure that the extensions of those blocks intersect in exactly three varieties. This is a contradiction as  $9 + 18$  is not less than or equal to 25.  $\square$

### REFERENCES

- [1] R.H.F. Denniston, **Enumeration of Symmetric Designs (25, 9, 3)**, Annals of Discrete Math. 15 (1982), 111-127.
- [2] R. Mathon, private communication.
- [3] R. Mathon and A. Rosa, **Tables of Parameters of BIBD's with  $r \leq 41$  Including Existence, Enumeration and Resolvability Results**, Annals of Discrete Math. 265 (1985), 275-308.
- [4] R. Mathon and A. Rosa, **A Census of Mendelsohn Triple Systems of Order 9**, Ars Combinatoria 4 (1977), 309-315.
- [5] E.J. Morgan, **Some Small Quasi-Multiple Designs**, Ars Combinatoria 3 (1977), 233-250.