The CRC Handbook of Combinatorial Designs

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1 Lotto Designs

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1.1 Definitions, Examples and Remarks

1.1 Definition An (n, k, p, t)-*lotto design* is an n-set, V, of elements and a set $\mathcal{B}$ of k-element subsets (blocks) of V, so that for any p-subset P of V, there is a block $B \in \mathcal{B}$, for which $|P \cap B| \geq t$. $L(n, k, p, t)$ denotes the smallest number of blocks in any (n, k, p, t)-lotto design.

1.2 Remark There are many lotteries in the world run by governments and casinos. Generally, for a small fee, a person chooses k numbers from n numbers or has the numbers chosen randomly for her/him. This is the ticket. At a certain point, no more tickets are sold and the government or casino picks p numbers from the n numbers, in some random fashion. These are called the winning numbers. If any of the sold tickets match t or more of the winning numbers, then a prize is given to the holder of the winning ticket. The larger the value of t (usually $t \geq 3$) , the larger the prize. Many gamblers and researchers are interested to know what is the minimum number of tickets necessary to ensure a match of at least t numbers. This minimum is $L(n, k, p, t)$. Buying $L(n, k, p, t)$ tickets guarantees a minimum return but does not in any way change the expected return.

1.3 Remark Of course, $L(n, k, p, 1) = \lceil \frac{n-p+1}{k} \rceil$ and $L(n, k, k, k) = \binom{n}{k}$. A few other lottery's numbers are known. The Hungarian Lottery is a (90,5,5,t)-lotto design and $L(90, 5, 5, 2) = 100$. The lotteries with parameters (49,6,6,t) are popular with governments all over the world and it is known that $L(49, 6, 6, 2) = 19$. But many of these numbers are very difficult to obtain. For example, $L(49, 6, 6, 3)$ is not known, although it is known that $87 \leq L(49, 6, 6, 3) \leq 163$. The upper bound is obtained by adjoining the 77 blocks of the Steiner System, S(3,6,22) to the 86 blocks of a (27,6,4,3)-lotto design that was found using simulated annealing. The lower bound is obtained by counting.

1.4 Example The following three blocks form a (7,5,4,3)-lotto Design with the fewest number of blocks.
{1,2,3,4,7}, {1,2,5,6,7}, {3,4,5,6,7}.

1.5 Example The following six blocks form a (20,10,6,4)-lotto Design with the fewest number of blocks.
{1,3,5,6,7,10,11,12,13,14}, {2,4,5,6,7,8,9,10,12,15},
{1,2,3,4,8,9,11,13,14,15}, {3,5,6,10,12,16,17,18,19,20},
{1,7,9,11,14,16,17,18,19,20}, {2,4,8,13,15,16,17,18,19,20}.

1.6 Theorem If the blocks of a lotto design are complemented, a lotto design results so $L(n, k, p, t) = L(n, n-k, n-p, n-k-p+t)$.

1.2 Connections Between Lotto Designs and Other Designs

1.7 Definition A $t-(v,k,m,\lambda)$ *general cover* is a v-set V of elements and a set $\mathcal{B}$ of k-element subsets of V (blocks), so that for any m-subset M of V, there are at least λ blocks B of $\mathcal{B}$ for which $|M \cap B| \geq t$. $C_\lambda(v,k,m,t)$ is the smallest number of blocks in a $t-(v,k,m,\lambda)$ general cover. See §IV.**??** for examples and tables.

1.8 Proposition An (n,k,p,t)-lotto design is a $t-(n,k,p,1)$ general cover, so $L(n,k,p,t) = C_1(n,k,p,t)$.

1.9 Definition An (n,k,t) *covering design* or *cover* is an n-set X of elements and a set $\mathcal{B}$ of k-element subsets of X (blocks), so that every t-subset of X occurs in a block of $\mathcal{B}$. $C(n,k,t)$ is the smallest number of blocks in a (n,k,t) covering design. See §IV.**??** for examples and tables.

1.10 Proposition Since an (n,k,t,t)-lotto design is a (n,k,t) covering design, $L(n,k,t,t) = C(n,k,t)$.

1.11 Definition An (n,p,t) *Turán design* is an n-set X of elements and a set $\mathcal{B}$ of t-element subsets of X (blocks), so that every p-subset of X contains a block of $\mathcal{B}$. $T(n,p,t)$ is the smallest number of blocks in a (n,p,t) Turán design. See §IV. **??**.

1.12 Proposition An (n,t,p,t)-lotto design is a (n,p,t) Turán design, so $L(n,t,p,t) = T(n,p,t)$.

1.13 Remark If the blocks of a covering design are complemented then one gets a Turán design, so $C(n,k,t) = T(n,n-t,n-k)$. This is a just special case of Theorem 1.6

1.14 Definition Let $\mathcal{V}$ be a non-empty set of binary vectors(codewords), each of length n and each having weight k (i.e. k 1s and $n-k$ 0s). $\mathcal{V}$ is an (n,k,p,d)-*constant weight covering code* if every binary vector of weight p and length n has Hamming distance at most d from at least one codeword. $K(n,k,p,d)$ is the smallest number of codewords in an (n,k,p,d)-constant weight covering code.

1.15 Proposition Assuming $d \equiv k+p \pmod 2$, then $K(n,k,p,d) = L(n,k,p,(k+p-d)/2)$.

1.3 Theorems on Lotto Designs

1.16 Theorem

$$\frac{\binom{n}{k}}{\binom{p-1}{t-1}\binom{k}{t}} \cdot \frac{n-p+1}{n-t+1} \leq L(n,k,p,t) \leq \left\lceil \frac{\min(\binom{n}{p},\binom{n}{t})}{\lfloor \frac{k}{t} \rfloor} \right\rceil$$

1.17 Lemma (volume bound)

$$L(n,k,p,t) \geq \frac{\binom{n}{p}}{\sum_{i=t}^{\min(k,p)} \binom{k}{i}\binom{n-k}{p-i}}$$

1.18 Theorem $L(n,k,p,t)$ is a non-decreasing function under any one of the following conditions: if n increases, k decreases, p decreases or t increases. $L(n,k,p,t)$ is a non-decreasing function under any one of the following conditions: if n and k both decrease,n and p both decrease, k and t both increase or p and t both increase. $L(n,k,p,t)$ is a non-decreasing function if n,k,p,t all increase.

1.19 Theorem [4] For all $1 \leq t \leq \{k, p\} \leq n$
a) $L(n, k, p, t) = 1$, if and only if $k + p \geq n + t$.
b) $L(n, k, p, t) = 2$ if and only if $2t - 1 + \max\{n - 2k, 0\} \leq p \leq n + t - k - 1$.
c) $L(n, k, p, t) = 3$ if and only if $p \leq \min\{2t - 2 + \max\{n - 2k, 0\}, n - k + t - 1\}$ and

$$p \geq \begin{cases} 3t - 2 + \max\{n - 3k, 0\}, & \text{if } n \geq 2k, \\ \frac{3}{2}t - 1 + \max\{n - \frac{3}{2}k, 0\}, & \text{if } n < 2k. \end{cases}$$

1.20 Remark For $t = 1$, $L(n, k, p, t)$ has been completely determined. For t=2, the determination of $L(n, k, p, t)$ is closely related to finding the largest independent set in a multigraph. Using this, many families of infinite cardinality have been determined for $t = 2$ and the bounds on $L(n, k, p, 2)$ are quite good. For $t \geq 3$, the situation is much more open. Independent sets in graphs are also useful for general t.

1.21 Theorem [7] $L(n, k, p, 2) \geq \frac{n(n-p+1)}{k(k-1)(p-1)}$ with equality if an $S(2, k, s)$ exists with $n = s(p - 1)$, where s is an integer.

1.22 Theorem [3]

$$\begin{aligned} L(2m+1, 3, 3, 2) &= C(m, 3, 2) + C(m+1, 3, 2), \\ L(4m+2, 3, 3, 2) &= C(2m+1, 3, 2) + C(2m+1, 3, 2), and \\ L(4m, 3, 3, 2) &= C(2m-1, 3, 2) + C(2m+1, 3, 2). \end{aligned}$$

1.23 Theorem [6]

$$\min_{\sum_{i=1}^{p-1} \alpha_i = n} \sum_{i=1}^{p-1} \alpha_i \left\lceil \frac{\alpha_i - 1}{k - 1} \right\rceil \leq L(n, k, p, 2) \leq \min_{\sum_{i=1}^{p-1} \alpha_i = n} \sum_{i=1}^{p-1} C(\alpha_i, k, 2)$$

1.24 Theorem [1] For $6 \leq n \leq 30, L(n, 6, 6, 2) = \lceil \frac{n-1}{2} \rceil$.

n	31-33	34	35-36	37	38-39	40	41-42	43	44-45	46	47	48	49-50	51	52	53	54
$L(n, 6, 6, 2)$	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23

1.25 Theorem [8](Generalized Schönheim Bound) For $n \geq 2k - t + 1$ and $n \geq k - t + p + 1$,

$$L(n, k, p, t) \geq \left\lceil \frac{\binom{n}{k-t+1} L(n - k + t - 1, k, p, t)}{\binom{n}{k-t+1} - \binom{k}{k-t+1}} \right\rceil.$$

For $L(n, k, t, t)$ this bound reverts to the famous Schönheim Bound. See §IV.**??** for examples and tables.

1.26 Proposition [8] $L(n, k, t + 1, t) \geq \min L(n, k, t - 1, t - 1), L(n - t + 1, k - t + 2, 2, 2)$. There are many similar lower bounds.

1.27 Proposition If $k_1 > k_2$ then, $L(n, k_1, p, t) \geq \frac{L(n, k_2, p, t)}{L(k_1, k_2, t, t)}$.

1.28 Proposition [8] If $n - k \geq p - t + 1$ and $\lceil \frac{r}{r+1} n \rceil \leq k$ where $r = \left\lfloor \frac{p}{p-t+1} \right\rfloor$, then $L(n, k, p, t) = r + 1$.

1.29 Proposition [8] If $k \geq 3$, then $L(2k + 1, k, 5, 3) = 5$.
If $k \geq 3$, then $L(3k + 2, k, 8, 3) = 5$.
if $k \geq 4$, then $L(2k + 2, k, 8, 4) = 4$.

1.30 Proposition [8] If there exists an element of frequency 0 in an (n, k, p, t) lotto design with $y > 1$ blocks, then there exists an $(n-1, k, p-1, t)$-lotto design on y blocks where $n \geq k+1$.

1.31 Proposition [8] If there exists an element of frequency 1 in an (n, k, p, t)-lotto design with $y > 1$ blocks, then there exists an $(n-k+t-2, k, p-1, t)$-lotto design with y-1 blocks where $n > 2k-t+1$ and $t > 1$.

1.4 Constructions, Algorithms and Tables

1.32 Proposition If $n = n_1 + n_2$ and $p = p_1 + p_2 - 1$, then $L(n, k, p, t) \leq L(n_1, k, p_1, t) + L(n_2, k, p_2, t)$.

1.33 Proposition $L(n+1, k+1, p+1, t+1) \leq L(n, k, p, t) + L(n, k+1, p+1, t+1)$.

1.34 Proposition [8](Semi-direct Product) Suppose n, k, p, t, n_1, k_1 and r are integers such that $n_1 < n$, $p - r \geq t, k_1 \geq t - r - 1$ and $k_1 = k - n + n_1$. Then $L(n, k, p, t) \leq L(n_1, k, p-r, t) + L(n_1, k_1, p-r-1, t-r-1)$.

1.35 Construction [2] Let $n = a+b+c$ where $a \geq 3, c \geq 3, a+b \geq 6, b+c \geq 6$ and $m = 5$ or 6. Then $L(n, 6, m, 4) \leq L(a+b, 6, m-1, 4) + L(b+c, 6, m-1, 4) + L(a, 3, 2, 2)L(c, 3, 2, 2)$.

1.36 Remarks Another way to improve the bounds is to use computer programs. The lower bounds can be improved by running backtrack algorithms. Unfortunately, they are very slow and useful only if the number of blocks is very small. The upper bounds can often be improved by heuristic algorithms. The most successful heuristic algorithm used is simulated annealing, although tabu search, integer linear programming, greedy algorithms and other algorithms have found some upper bounds. See §VI.9 **??** for descriptions of these algorithms.

1.37 Table $L(n, k, p, t)$ for $t = 2, 3, 4$; $p = t+1, \ldots, 11$; $k = t+1, \ldots, 14$ and $n = p+1, \ldots, 20$. If the value is unknown, upper and lower bounds are given. Since Theorem 1.19 determines when the number of blocks is 1, 2 or 3, any row with just these numbers will be left out. Also $L(n, k, t, t)$ entries were left out as they are recorded in the tables on coverings (See §IV.**??**). For more tables on lotto designs, see www.cs.umanitoba.ca/ lipakc/lottotables.html, http://dip.sun.ac.za/ vuuren/repositories/lottery/repository.php or www.xs4all.nl/∼rbelic.

$p = 3$, $t = 2$

$n \backslash k$	3	4	5	6	7	8	9	10	11	12	13	14
7	4	2	2	1	1							
8	5	2	2	2	1	1						
9	7	4	2	2	2	1	1					
10	8	4	2	2	2	2	1	1				
11	10	6	4	2	2	2	2	1	1			
12	11	6	4	2	2	2	2	2	1	1		
13	13	8	5	4	2	2	2	2	2	1	1	
14	14	9	6	4	2	2	2	2	2	2	1	1
15	18	11	7	4	4	2	2	2	2	2	2	1
16	19	12	8	5	4	2	2	2	2	2	2	2
17	23	14	9	6	4	4	2	2	2	2	2	2
18	24	15	10	6	5	4	2	2	2	2	2	2
19	29	16	11	7	6	4	4	2	2	2	2	2
20	31	18	12	8	6	4	4	2	2	2	2	2

$p = 4$, $t = 2$

$n \backslash k$	3	4	5	6	7	8	9	10	11	12	13	14
10	5	3	2	2	2	1	1	1				
11	6	3	2	2	2	2	1	1	1			
12	8	3	3	2	2	2	2	1	1	1		
13	9	5	3	2	2	2	2	2	1	1	1	
14	11	5	3	3	2	2	2	2	2	1	1	1
15	12	7	3	3	2	2	2	2	2	2	1	1
16	14	7	5	3	3	2	2	2	2	2	2	1
17	15	9	5	3	3	2	2	2	2	2	2	2
18	17	9	6	3	3	3	2	2	2	2	2	2
19	18	11	7	5	3	3	2	2	2	2	2	2
20	20	12	8	5	3	3	3	2	2	2	2	2

$p = 5$, $t = 2$

$n \backslash k$	3	4	5	6	7	8	9	10	11	12	13	14
11	4	3	2	2	2	1	1	1	1			
12	4	3	2	2	2	2	1	1	1	1		
13	6	3	3	2	2	2	2	1	1	1	1	
14	7	4	3	2	2	2	2	2	1	1	1	1
15	9	4	3	3	2	2	2	2	2	1	1	1
16	10	4	3	3	2	2	2	2	2	2	1	1
17	12	6	4	3	3	2	2	2	2	2	2	1
18	13	6	4	3	3	2	2	2	2	2	2	2
19	15	8	4	3	3	3	2	2	2	2	2	2
20	16	8	4	4	3	3	2	2	2	2	2	2

$p = 6,\, t = 2$

$n\backslash k$	3	4	5	6	7	8	9	10	11	12	13	14
12	4	3	2	2	2	1	1	1	1	1		
13	4	3	2	2	2	2	1	1	1	1	1	
14	5	3	3	2	2	2	2	1	1	1	1	1
15	5	4	3	2	2	2	2	2	1	1	1	1
16	7	4	3	3	2	2	2	2	2	1	1	1
17	8	4	3	3	2	2	2	2	2	2	1	1
18	10	5	4	3	3	2	2	2	2	2	2	1
19	11	5	4	3	3	2	2	2	2	2	2	2
20	13	5	4	3	3	3	2	2	2	2	2	2

$p = 7,\, t = 2$

$n\backslash k$	3	4	5	6	7	8	9	10	11	12	13	14
13	4	3	2	2	2	1	1	1	1	1	1	
14	4	3	2	2	2	2	1	1	1	1	1	1
15	5	3	3	2	2	2	2	1	1	1	1	1
16	5	4	3	2	2	2	2	2	1	1	1	1
17	6	4	3	3	2	2	2	2	2	1	1	1
18	6	4	3	3	2	2	2	2	2	2	1	1
19	8	5	4	3	3	2	2	2	2	2	2	1
20	9	5	4	3	3	2	2	2	2	2	2	2

$p = 8,\, t = 2$

$n\backslash k$	3	4	5	6	7	8	9	10	11	12	13	14
14	4	3	2	2	2	1	1	1	1	1	1	1
15	4	3	2	2	2	2	1	1	1	1	1	1
16	5	3	3	2	2	2	2	1	1	1	1	1
17	5	4	3	2	2	2	2	2	1	1	1	1
18	6	4	3	3	2	2	2	2	2	1	1	1
19	6	4	3	3	2	2	2	2	2	2	1	1
20	7	5	4	3	3	2	2	2	2	2	2	1

$p = 9,\, t = 2$

$n\backslash k$	3	4	5	6	7	8	9	10	11	12	13	14
15	4	3	2	2	2	1	1	1	1	1	1	1
16	4	3	2	2	2	2	1	1	1	1	1	1
17	5	3	3	2	2	2	2	1	1	1	1	1
18	5	4	3	2	2	2	2	2	1	1	1	1
19	6	4	3	3	2	2	2	2	2	1	1	1
20	6	4	3	3	2	2	2	2	2	2	1	1

$p = 10,\, t = 2$

$n\backslash k$	3	4	5	6	7	8	9	10	11	12	13	14
16	4	3	2	2	2	1	1	1	1	1	1	1
17	4	3	2	2	2	2	1	1	1	1	1	1
18	5	3	3	2	2	2	2	1	1	1	1	1
19	5	4	3	2	2	2	2	2	1	1	1	1
20	6	4	3	3	2	2	2	2	2	1	1	1

$p = 11$, $t = 2$

$n\backslash k$	3	4	5	6	7	8	9	10	11	12	13	14
17	4	3	2	2	2	1	1	1	1	1	1	1
18	4	3	2	2	2	2	1	1	1	1	1	1
19	5	3	3	2	2	2	2	1	1	1	1	1
20	5	4	3	2	2	2	2	2	1	1	1	1

$p = 4$, $t = 3$

$n\backslash k$	4	5	6	7	8	9	10	11	12	13	14
7	4	3	1	1							
8	6	3	3	1	1						
9	9	5	3	3	1	1					
10	12-14	7	4	3	3	1	1				
11	16-19	9	5	3	3	3	1	1			
12	21-26	11-12	6	4	3	3	3	1	1		
13	28-33	11-16	8-9	6	4	3	3	3	1	1	
14	35-42	14-20	8-11	6	5	3	3	3	3	1	1
15	44-52	18-24	10-14	7-9	6	4	3	3	3	3	1
16	54-64	22-32	11-16	7-11	7	5	4	3	3	3	3
17	66-77	27-40	14-20	9-13	7-9	6	4	3	3	3	3
18	79-93	32-48	16-25	10-15	7-9	6	5	4	3	3	3
19	94-110	38-57	19-30	11-18	9-14	7-9	6	5	4	3	3
20	111-128	45-70	23-35	13-20	9-14	7-10	6-8	5	4	3	3

$p = 5$, $t = 3$

$n\backslash k$	4	5	6	7	8	9	10	11	12	13	14
9	5	2	2	1	1	1					
10	7	2	2	2	1	1	1				
11	10	5	2	2	2	1	1	1			
12	10-12	6	2	2	2	2	1	1	1		
13	13	8	5	2	2	2	2	1	1	1	
14	17-20	8-10	5	2	2	2	2	2	1	1	1
15	21-26	9-13	7	5	2	2	2	2	2	1	1
16	26-28	11-14	7-8	5	2	2	2	2	2	2	1
17	32-39	13-18	7-11	6-7	5	2	2	2	2	2	2
18	39-44	16-22	8-12	7-8	5	2	2	2	2	2	2
19	46-55	19-26	10-15	7-10	6	5	2	2	2	2	2
20	54-60	22-32	11-18	7-11	6-7	5	2	2	2	2	2

$p = 6$, $t = 3$

$n\backslash k$	4	5	6	7	8	9	10	11	12	13	14
10	4	2	2	1	1	1	1				
11	5	2	2	2	1	1	1	1			
12	7	4	2	2	2	1	1	1	1		
13	10	4	2	2	2	2	1	1	1	1	
14	10-12	6	4	2	2	2	2	1	1	1	1
15	13-15	8	4	2	2	2	2	2	1	1	1
16	16-20	8-10	5	4	2	2	2	2	2	1	1
17	19-23	8-12	6	4	2	2	2	2	2	2	1
18	23-28	10-14	7	4	4	2	2	2	2	2	2
19	28-33	11-17	7-9	5	4	2	2	2	2	2	2
20	33-40	13-20	7-10	6-7	4	4	2	2	2	2	2

$p = 7,\ t = 3$

$n\backslash k$	4	5	6	7	8	9	10	11	12	13	14
13	6	3	2	2	2	1	1	1	1	1	
14	8	3	2	2	2	2	1	1	1	1	1
15	9-10	3	3	2	2	2	2	1	1	1	1
16	10-12	6	3	2	2	2	2	2	1	1	1
17	12-14	7	3	3	2	2	2	2	2	1	1
18	14-17	7-9	3	3	2	2	2	2	2	2	1
19	17-19	8-11	6	3	3	2	2	2	2	2	2
20	20-25	8-13	6	3	3	2	2	2	2	2	2

$p = 8,\ t = 3$

$n\backslash k$	4	5	6	7	8	9	10	11	12	13	14
14	5	3	2	2	2	1	1	1	1	1	1
15	6	3	2	2	2	2	1	1	1	1	1
16	7-8	3	3	2	2	2	2	1	1	1	1
17	9-11	5	3	2	2	2	2	2	1	1	1
18	9-13	5	3	3	2	2	2	2	2	1	1
19	11-16	7	3	3	2	2	2	2	2	2	1
20	13-17	7-9	5	3	3	2	2	2	2	2	2

$p = 9,\ t = 3$

$n\backslash k$	4	5	6	7	8	9	10	11	12	13	14
15	4	3	2	2	2	1	1	1	1	1	1
16	4	3	2	2	2	2	1	1	1	1	1
17	7	3	3	2	2	2	2	1	1	1	1
18	7-9	4	3	2	2	2	2	2	1	1	1
19	8-11	4	3	3	2	2	2	2	2	1	1
20	10-13	4	3	3	2	2	2	2	2	2	1

$p = 10,\ t = 3$

$n\backslash k$	4	5	6	7	8	9	10	11	12	13	14
16	4	3	2	2	2	1	1	1	1	1	1
17	4	3	2	2	2	2	1	1	1	1	1
18	6	3	3	2	2	2	2	1	1	1	1
19	7	4	3	2	2	2	2	2	1	1	1
20	8-9	4	3	3	2	2	2	2	2	1	1

$p = 11,\ t = 3$

$n\backslash k$	4	5	6	7	8	9	10	11	12	13	14
17	4	3	2	2	2	1	1	1	1	1	1
18	4	3	2	2	2	2	1	1	1	1	1
19	5	3	3	2	2	2	2	1	1	1	1
20	5	4	3	2	2	2	2	2	1	1	1

$p = 5,\ t = 4$

$n\backslash k$	5	6	7	8	9	10	11	12	13	14
8	5	3	1	1						
9	9	3	3	1	1					
10	10-14	7	3	3	1	1				
11	17-22	8-10	5	3	3	1	1			
12	26-35	10-14	8	3	3	3	1	1		
13	37-48	13-21	9-10	6	3	3	3	1	1	
14	52-69	18-31	9-14	7	5	3	3	3	1	1
15	71-95	24-40	11-20	7-11	7	3	3	3	3	1
16	95-132	32-54	14-28	7-14	7-9	5	3	3	3	3
17	124-175	42-70	18-40	9-20	7-12	7	5	3	3	3
18	159-215	53-81	23-51	12-30	7-16	7-9	6-7	3	3	3
19	202-285	68-115	29-66	15-38	8-23	7-14	6-9	5	3	3
20	252-359	84-142	36-75	18-42	10-29	7-15	6-12	6-7	5	3

$p = 6,\ t = 4$

$n\backslash k$	5	6	7	8	9	10	11	12	13	14
9	4	3	1	1	1					
10	7	3	3	1	1	1				
11	8-10	5	3	3	1	1	1			
12	11-16	6	4	3	3	1	1	1		
13	15-23	8-10	5	3	3	3	1	1	1	
14	21-34	8-14	7	4	3	3	3	1	1	1
15	29-49	10-19	7-9	6	4	3	3	3	1	1
16	38-65	13-25	7-14	6	5	3	3	3	3	1
17	50-68	17-34	9-19	7-9	6	4	3	3	3	3
18	64-106	22-42	10-24	7-12	6-7	5	4	3	3	3
19	81-144	27-54	12-29	9-17	7-9	6	4	3	3	3
20	101-198	34-66	15-38	9-20	7-12	6	5	4	3	3

$p = 7,\ t = 4$

$n\backslash k$	5	6	7	8	9	10	11	12	13	14
11	5	2	2	1	1	1	1			
12	8	2	2	2	1	1	1	1		
13	8-12	5	2	2	2	1	1	1	1	
14	11-18	6	2	2	2	2	1	1	1	1
15	15-26	8-10	5	2	2	2	2	1	1	1
16	20-36	8-14	6	2	2	2	2	2	1	1
17	26-50	9-19	6-9	5	2	2	2	2	2	1
18	34-60	12-24	6-12	5	2	2	2	2	2	2
19	43-80	15-32	7-16	6-8	5	2	2	2	2	2
20	54-96	18-40	8-20	6-10	5	2	2	2	2	2

$p = 8,\ t = 4$

$n\backslash k$	5	6	7	8	9	10	11	12	13	14
12	4	2	2	1	1	1	1	1		
13	6	2	2	2	1	1	1	1	1	
14	8-10	4	2	2	2	1	1	1	1	1
15	8-13	4	2	2	2	2	1	1	1	1
16	10-18	7	4	2	2	2	2	1	1	1
17	14-23	7-10	4	2	2	2	2	2	1	1
18	17-31	7-14	6	4	2	2	2	2	2	1
19	22-42	8-17	6-9	4	2	2	2	2	2	2
20	27-52	9-21	6-11	4	4	2	2	2	2	2

$p = 9,\ t = 4$

$n\backslash k$	5	6	7	8	9	10	11	12	13	14
13	4	2	2	1	1	1	1	1	1	
14	5	2	2	2	1	1	1	1	1	1
15	8	4	2	2	2	1	1	1	1	1
16	8-10	4	2	2	2	2	1	1	1	1
17	9-14	6	4	2	2	2	2	1	1	1
18	10-18	6	4	2	2	2	2	2	1	1
19	13-23	7-10	5	4	2	2	2	2	2	1
20	16-28	7-13	6	4	2	2	2	2	2	2

$p = 10,\ t = 4$

$n\backslash k$	5	6	7	8	9	10	11	12	13	14
16	6	3	2	2	2	1	1	1	1	1
17	7-9	3	2	2	2	2	1	1	1	1
18	7-12	3	3	2	2	2	2	1	1	1
19	9-15	6	3	2	2	2	2	2	1	1
20	11-19	7	3	3	2	2	2	2	2	1

$p = 11,\ t = 4$

$n\backslash k$	5	6	7	8	9	10	11	12	13	14
17	5	3	2	2	2	1	1	1	1	1
18	7	3	2	2	2	2	1	1	1	1
19	7-10	3	3	2	2	2	2	1	1	1
20	8-13	5	3	2	2	2	2	2	1	1

1.38 Table [2] For $21 \leq n \leq 32$ and $k = 6$, we list a selection of upper bounds for small $p - t$.

$n \backslash p, t$	4,3	5,3	6,3	7,3	5,4	6,4	7,4	6,5	7,5
21	40	21	13	9	169	80	52	1124	497
22	46	22	15	9	189	101	61	1456	632
23	54	26	17	11	229	127	73	1903	888
24	61	30	20	12	267	143	82	2402	1196
25	68	36	22	15	335	175	107	3043	1513
26	76	40	25	16	403	220	121	3832	1903
27	86	46	27	19	484	243	146	4800	2406
28	98	49	31	22	595	297	160	6191	3058
29	108	56	35	23	719	333	197	7675	3755
30	119	61	39	26	874	397	232	9027	4275
31	133	68	45	29	1028	442	268	11063	5373
32	149	73	50	32	1217	540	304	12823	6560

1.5 See Also

§IV.??	General Covers with $\lambda = 1$ are equivalent to lotto designs.
§IV.??	Covering Designs and Turán Designs are special cases of lotto designs 1.10 and 1.12
§V.??	Constant weight covering codes are lotto designs 1.15
[5]	Constructions of constant weight covering codes and applications to quantizers.

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